OGO GAS BUDGET COMPUTATION: DESCRIPTION OF PROGRAM AND PRESENTATION OF COMPUTED GAS BUDGETS

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by

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PREFACE

The work described in this report was performed under NASA Contract No. NAS-5-5742, Work Order No. 620-W-35101, and was monitored by Mr. Gil Fleisher, Code No. 622. The work at BAARINC was under the administrative direction of E. J. Bacon, Research Director, and was technically performed by J. Currier, R. Saaty, and S. Harrison, under the latter's direction.

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SUMMARY

The digital computer program described in this report was written for the purpose of estimating POGO and EGO attitude control gas budgets. In contrast to one previously developed by STL,* the present program specifically includes a detailed simulation of booms, experimental packages and antennae, together with their shadowing by the spacecraft body and solar paddles. The object of the present program was to check the STL gas budget computations on the assumption that these latter might be unduly optimistic.

This suspicion was found to be well grounded. For a proposed POGO initial orbital of 150 n. m. perigee, it was found that the total gas available would be exhausted within about 2 months of the launch epoch. Boosting the initial perigee to 180 n. m. and 200 n. m. raised the lifetime to about 4 months and 7 months, respectively. Complete elimination of aerodynamic torques gave a life of only 11 months; hence, evidently, no raising of initial perigee height, within reasonable limits, will extend the lifetime to a year.

* D. D. Otten, "OGO Attitude Control Subsystem Description, Logic, and Specifications," Space Technology Laboratories Inc., 2313-0004-RU-000, December 1961.

The program takes into account torques due to aerodynamic and solar pressures and gravity gradient. Analysis of the output data revealed that aerodynamic yaw torque was the major cause of gas expenditure up to an initial perigee altitude of about 200 n. m., i. e., within the range of interest for POGO. Further analysis disclosed that most of this torque was due to the unbalancing effects of the EP5 torus and the SOEP VLF antenna. At NASA's suggestion, POGO flights were simulated with these two antennae undeployed, both singly and in combination. At 180 n. m. suppression of the deployment of both antennae doubled the satellite's oriented lifetime; the corresponding improvement at 200 n. m. was, of course, somewhat less, due to the thinner atmosphere.

The only other major source of torque was gravity-gradient yaw.

Some attempt was made to identify sources of error and bias in these lifetime figures and to assess their order of magnitude. A major weakness is the uncertainty of atmospheric density as a function of height; a second serious bias concerns uncertainties in the value of the aerodynamic reflection coefficients. Two major weaknesses in the program itself are the omission of coulomb drag effects (which, it is demonstrated, may be considerable, due to POGO's high projected perimeter) and the inability to follow true yaw angle during an eclipse.

It appeared that the joint effects of these and other uncertainties could impose a fourfold error upon computed gas budgets.

The possibility of ~~co~~rrecting torque imbalance by the addition of compensating "sail" surfaces to the satellite, and hence prolonging satellite life, was raised by NASA. This remedy was examined and shown to be vulnerable to errors in the estimates of aerodynamic reflection coefficients.

EGO gas budget computations contrast very favorably with those of POGO. It was found that a year in orbit consumed only about 150 pound-seconds of gas. Further, this estimate is not subject to the same errors as those of POGO. The satellite is in the atmosphere for only a small fraction of the orbital period during early orbits; increases in perigee altitude lift the entire orbital out of the atmosphere within a few weeks after the launch epoch. Hence, EGO torques are largely due to solar pressure and gravity gradient, both of which can be computed with reasonable accuracy.

A parametric analysis was made of gas budget dependence upon orbital inclination to the sun vector, and the angular position of perigee relative to the projection of the sun vector onto the orbital plane. At the same time, some attempt was made to rationalize these values by a deductive examination of the effects of orbital orientation upon torque magnitudes, careful distinctions being made between secular (i.e., cumulative) and cyclic disturbance torques.

The program assumes that attitude control functions normally; this permitted dynamic simulation of the satellite to be omitted. As a consequence, gas consumption cannot be followed historically, in terms of discrete gas firings, but only macroscopically, in terms of the amount of impulse required to unload a total accumulation of angular momentum. The program computes gas expenditure per orbital cycle. Total expenditure over a 12-month period is estimated by sampling orbits at intervals throughout. Gas budgets reported herein were based upon sampling orbitals at 15-day intervals. Orbital perturbations are allowed for by adjusting parameters of each orbital. These adjustments included precession of perigee, regression of line of nodes, movement of the sun vector, changes in eclipse angles, and changes in perigee height and orbital eccentricity. All of these can be obtained analytically, and were so obtained, except for the last two, which were supplied by NASA for selected POGO and EGO flights.

An appendix has been added as a convenience, describing how initial orbital parameters may be computed from the injection parameters.

I. DESCRIPTION OF COMPUTER PROGRAM

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1. INTRODUCTION

An outline of the program is presented in the flow chart at the end of this section.

The program accepts inputs for a number of different orbits representing discrete samples at intervals in the orbital history of the satellite throughout the year.

Each orbital revolution is broken down into a number of equal intervals (this number being an input to the program) and torques computed for aerodynamic, solar, and gravity-gradient for each increment around the orbital. Any combination of these three categories may be suppressed by suitable input designations.

Torques are converted into torque impulses (= angular momenta) and dumped into an inertial coordinate system. All control torques (which are not computed in this program) are assumed to cancel. Cyclic components of disturbance torques computed by the program will automatically self-cancel as they are dumped into the inertial system.

The total angular momentum for the x, y, and z coordinates obtained for each orbit is converted into a gas expenditure (in pound-seconds) for that orbit. This figure is multiplied by the number of orbits occurring in that sampling interval to obtain the gas expenditure for the corresponding real-time interval. Finally, these gas expenditures are cumulated for successive orbital intervals to give the total gas budget requirement for the satellite lifetime.

Detailed printouts are available on demand for each orbit. In the absence of such demand, these outputs are suppressed.

The EGO spacecraft has minor structural differences compared to POGO (chiefly with respect to the angular orientation of the EP5 torus). Also, the more eccentric orbit calls for different sampling intervals around the orbital. These two modifications are controlled by a POGO/EGO switch which is set by an input card as required.

Notations and subroutine details are presented in separate sections below.

2. DEFINITIONS OF ORBIT PARAMETERS

μ = GM (gravitational constant G times mass of earth M) $= 1.408 \times 10^{16}$ ft.^{3}/sec.^{2} in the English system

r_e = Radius of earth = 2.0902913×10^7 feet

a = Semimajor axis of ellipse in feet

e = Eccentricity of orbit

ξ = Inclination of orbit plane from ecliptic plane

Ω = Angle from line of nodes to perigee (in orbit plane)

S = Angle of sun vector from equinox line in ecliptic plane. Use autumnal equinox, i. e., negative of vernal equinox, for reference line.

β = Angle from vernal equinox to line of nodes (in ecliptic plane).

The penumbra and umbra angles are:

α_1 = Entry into penumbra

α_2 = Entry into umbra and exit from first penumbra

α_3 = Exit from umbra and entry into second penumbra

α_4 = Exit from second penumbra.

(All these angles are given, in this order, as measured from perigee.)

S' = Angle of sun vector (declination) from perpendicular to orbit plane. The range is $0^\circ \leq S' \leq 180^\circ$.

α = Angle from line of nodes to satellite position

p = Period of orbit in seconds

t = Time elapsed in each sampling interval.

(1) Equations for Computing Orbit Parameters

The following equations are used for computing orbit parameters:

S' $\quad \cos S' = \sin \xi \sin (S-\beta)$, $\begin{Bmatrix} \cos S' > 0, & \text{quadrant } 1 \\ \cos S' < 0, & \text{quadrant } 2 \end{Bmatrix}$

p $\quad p = \dfrac{2\pi a^{3/2}}{\sqrt{\mu}}$

t $\quad t = \dfrac{p}{\text{number of intervals}}$.

(2) Method of Solution for Kepler's Equation

Kepler's equation is:

$$M = E - e \sin E, \tag{1}$$

where:

M	=	the mean anomaly angle (expressed in radians)
E	=	the eccentric anomaly angle (expressed in radians)
e	=	the eccentricity of the elliptical orbit.

The method of solution employed is an iterative technique given in Brouwer and Clemence.* The technique is as follows. M is known, and we wish to solve equation (1) for E. Choose a first guess for E, call it E_o (more will be said later about how to make this first choice). Then, replacing E by E_o in Kepler's equation, we have:

$$M_o = E_o - e \sin E_o . \tag{2}$$

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* Dirk Brouwer and Gerald Clemence, Methods of Celestial Mechanics, Academic Press, 1961, pp. 84-85.

Then, $\Delta E_o = E - E_o$ and $\Delta M_o = M - M_o$. Kepler's equation is a function of two variables and may be written

$$f(M, E) = -M + E - e \sin E = 0 . \tag{3}$$

Applying Taylor's formula for a function of two variables, * we expand $f(M, E)$ in a series about M_o and E_o:

$$f(M, E) = f(M_o, E_o) + \left\{ \frac{\partial f}{\partial M_o}(M - M_o) + \frac{\partial f}{\partial E_o}(E - E_o) \right\} + \ldots . \tag{4}$$

where the partial derivatives are evaluated at M_o and E_o. This yields

$$\begin{aligned} f(M, E) = {} & (-M_o + E_o - e \sin E_o) + \\ & \left\{ -1(M - M_o) + (1 - e \cos E_o)(E - E_o) \right\} + \ldots . \end{aligned} \tag{5}$$

From (2) the first term of (5) vanishes, so we have

$$f(M, E) = \left\{ -1(M - M_o) + (1 - e \cos E_o)(E - E_o) \right\} + \ldots . \tag{6}$$

Recall from (3) that $f(M, E) = 0$. Hence, we set (6) equal to zero and, neglecting the remaining terms of the Taylor series, obtain

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* See Wilfred Kaplan, Advanced Calculus, Addison-Wesley, 1952, p. 370.

$$E - E_o = (M - M_o) / (1 - e \cos E_o)$$

$$\text{or} \quad \Delta E_o = \Delta M_o / (1 - e \cos E_o). \tag{7}$$

Equation (7) is only an approximation. At this point the problem of convergence of the iterative process can be examined.

In general, the iterative process is as follows:

E_o = first guess

Step 1. $M_i = E_i - e \sin E_i$ $\quad i = 0, 1, \ldots . n$

Step 2. $\Delta M_i = M - M_i$ (where M is the known value of the mean anomaly angle)

Step 3. Is $\Delta M_i \leq$ error? When M_i is sufficiently close to the original M, the process is stopped and E_i is given as the solution.

Step 4. $\Delta E_i = \dfrac{\Delta M_i}{1 - e \cos E_i}$

Step 5. $E_{i+1} = E_i + \Delta E_i$

Go back to Step 1 and repeat the process with E_{i+1} for the $(i+1)^{th}$ iteration.

It may happen that the process does not give a convergent sequence tending to the solution. To prevent useless cycling in such a case, the number of iterations is limited to 25. Usually an error test of 0.0001 is used, and three or fewer iterations are sufficient when the eccentricity is small. In other words, when the process passes the error test, we know that the value of the eccentric anomaly substituted in Kepler's equation gives a value for the mean anomaly differing from the true mean anomaly by only 0.0001 radians (or 0.0057 degrees). This was considered sufficiently accurate for our purposes.

Method for making first guess. If the eccentricity is small, as it is in the case of POGO, it is sufficient to make a first guess $E_o = M$. In the case of EGO, a first guess of

$$E_o = M + e \sin M + \frac{1}{2} e^2 \sin 2M$$

will be used, as suggested in Brouwer and Clemence (p. 84). There is no indication of how well this will serve in the case of a nearly-parabolic orbit.

(3) Equations for Computing Interval Parameters

The following equations are used for computing interval parameters:

M -- Mean anomaly angle $\quad M_n = \frac{2\pi t_n}{p}$, for n^{th} time interval.

E -- Eccentric anomaly angle $\quad M = E - e \sin E$

Kepler's equation, solve by iterative procedure.

ν -- True anomaly angle $\quad \tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$

r -- Radius to center of earth $\quad r = a(1-e \cos E)$

V -- Velocity $\quad V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$

γ -- Flight path angle $\quad \tan \gamma = \frac{e \sin E}{\sqrt{1-e^2}}$

$$\cos \gamma = \left[\frac{a^2(1-e^2)}{r(2a-r)}\right]^{\frac{1}{2}}$$

Both equations necessary to fix quadrant of γ.

η -- Angle from ascending node to projection of sun vector $\quad \tan \eta = \frac{\cos \xi \sin (S-\beta)}{\cos (S-\beta)}$

$$\cos \eta = -\frac{\cos (S-\beta)}{\sqrt{\cos^2(S-\beta) + \cos^2 \xi \sin (S-\beta)}}$$

$$\sin \eta = -\frac{\cos \xi \sin (S-\beta)}{\sqrt{\cos^2(S-\beta) + \cos^2 \xi \sin^2 (S-\beta)}}$$

α -- Angle from ascending node to satellite $\quad \alpha = \Omega + \nu$.

The transformation matrix from body coordinate system (x_b, y_b, z_b) to inertial coordinate system (x_j, y_j, z_j) is as follows:

$$\begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} = \begin{bmatrix} \cos\psi\cos(\eta-\alpha) & -\sin\psi\cos(\eta-\alpha) & \sin(\eta-\alpha) \\ -\sin\psi & -\cos\psi & 0 \\ \cos\psi\sin(\eta-\alpha) & -\sin\psi\sin(\eta-\alpha) & -\cos(\eta-\alpha) \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix}$$

1. <u>Yaw Angle ψ</u>

$$\tan\psi = -\frac{\sin S'\sin(\alpha-\eta)}{\cos S'} .$$

Determination of quadrant:

$\sin(\alpha-\eta) < 0, \ \tan\psi > 0$, quadrant = 1

$\sin(\alpha-\eta) < 0, \ \tan\psi < 0$, quadrant = 2

$\sin(\alpha-\eta) \geq 0, \ \tan\psi > 0$, quadrant = 3

$\sin(\alpha-\eta) \geq 0, \ \tan\psi < 0$, quadrant = 4.

If $\cos S' = 0$, a special situation of "noon turn" applies. The sun in this case lies in the same plane as the orbit. Thus, when $\alpha-\eta = 0^{\circ}$ or 180°, ψ is rotating from 0° to 270° or from 270° to 0°, respectively. The torque encountered in this rotation is not taken into account as the control system is designed to compensate for these particular torques.

2. <u>Paddle Angle ϕ_p</u>

$$\sin\phi_p = -\sin S'\cos(\alpha-\eta) .$$

Determination of quadrant:

$\sin\phi_p > 0$, quadrant = 2

$\sin\phi_p < 0$, quadrant = 3.

(4) General Comments

This program was written in Fortran IV for the IBM 7094 computer and has been run on the Moonlight System at Goddard Space Flight Center. The data are input from cards and have been arranged with 10 to 20 free spaces at the beginning of each card (the exact number is indicated in the respective format statements). These spaces may be used for the operator's convenience in data identification since the program ignores them. A listing of the program is given at the end of this section and is followed by a sample data listing.

Each card of input data is described in detail under Program Inputs. The card number, variable names, interpretation of the variable names, and the format for the card are given. The data deck is made up of cards 1 to 44 followed by the proper number of sets of "type a" cards. Each set of "type a" cards gives the orbit parameters for one orbit.

(5) Program Inputs

Input data to the program are as follows:

Card No.	Variables	Interpretation	Format
1	NOSHAD, NEGO, NDAYS	NOSHAD and NEGO are options; see below. NDAYS is the number of days in a sampling interval. The orbit sampling process is described in the introduction.	(20X, 4I5)
2	FNØRB	Number of passes through perigee in a given orbit in NDAYS.	(10X, 6F10.3)
3	IAIR, ISUN, IGRAV	Options; see below.	(20X, 4I5)
4	ITØRTA	Option; see below.	(20X, 4I5)
5	F4y	Y-face of experiment four box	(10X, 6F10.3)
6	F4x	X-face of experiment four box	
7	CANT	High-gain antenna	(10X, 6F10.3)
8	S2	Sphere of experiment two	
9	C3	Cylinder of experiment three	
10	C1	Cylinder of experiment one	
11	BX	SOEP antenna	
12	COPEP	OPEP cylinder	

Card No.	Variables	Interpretation	Format
13	OPEP	--	
14	Boom 6 (B6)	Boom of experiment six	
15	Sphere 6 (S6)	Sphere of experiment six	
16	Boom 5 (B5)	Boom of experiment five	
17	CYLN 5 (C5)	Torus of experiment five	
18	Box-x5 (F5x)	X-face of experiment five box	
19	Box-y5 (F5y)	Y-face of experiment five box	
20	H, E, A, B, C, L (FL)	Body dimensions (see diagram in quoted reference)	
21	W, SGMAS (SGMA, SGMAP)	W is a body dimension; σ, σ' are previously defined	
22	Y, Z, OPEP, PAD(AY,AZ, AOP, AP)	Reflectivity constants for y-face, z-face, OPEP, and paddle	(20X, 4F10. 0)
23	B6, BX, B5, F5	Reflectivity constants for boom 6, etc.	(20X, 4F10. 0)
24	AC5	Reflectivity constant for torus	(20X, 4F10. 0)
25	OP	Areas of the three different faces of OPEP	(20X, 4F10. 0)
26 27 28 29 30	ATMO ↓	The atmospheric density look-up table. Starting with densities in slugs/ft.^{3} at 100 n. m. and extending to 750 n. m. in 50-n. m. steps.	(20X, 3E20. 8) ↓

Card No.	Variables	Interpretation	Format
31	V	Solar pressure constant	(20X, 3E20. 8)
32 33 34 35	THRSTX ↓	Look-up table for the thrust for the x and z momenta (see gas budget estimate)	(10X, 6F10. 3) ↓
36 37 38 39	THRSTY ↓	Look-up table for thrust around z-axis	(10X, 6F10. 3) ↓
40	XXI, YYI, ZZI	Moments of inertia about x, y, and z axes, respectively	(20X, 4F10. 0)
41	GTHX, GTHY, GTHZ	X, y, and z principal angles (from displacement of center of mass)	(20X, 4F10. 0)
42	NØRBIT, NINTER, IPRINT	NØRBIT is the total number of different orbits, each requiring a set of orbit parameters. NINTER is the number of intervals per orbit (NINTER $\leq$ 360). IPRINT is print option; see below.	(20X, 4I5)
43	ERRKEP	Test for solution of Kepler's equation by iterative technique (0. 0001 is a good choice)	(20X, 4F10. 0)
44*	GM, RE	GM is gravitational constant μ and has a value of 1.408×10^{16} ft.^{3}/sec.^{2} in	(16X, 4E16. 8)

* GM and RE are inputs so the program can compute in the c g s or k m s system of units as well as in the English system. Some minor changes to the final gas computations will make the program completely adaptable to any system of units.

Card No.	Variables	Interpretation	Format
		the English system. RE is the radius of the earth, and in the English system the value 2.0902913 x 10^7 ft. was used.	

One set of the following "type a" cards is needed for each orbit (generally around 25 orbits for a representative run), and the number of sets is given by the value of NØRBIT on card 42.

Card No.	Variables	Interpretation	Format
1a	A	Semimajor axis of orbit, in units of feet if the English system is being employed.	(20X, E16.8)
2a	E, XI, S	E is orbit eccentricity; XI is inclination of orbit plane from ecliptic plane (in degrees); and S is angle of sun vector from vernal equinox (in degrees).	(20X, 4F10.0)
3a	OMEGA, BETA	OMEGA is angle from line of nodes to perigee (in degrees), and BETA is angle from vernal equinox to apsides (in degrees).	(20X, 4F10.0)
4a	ALPHA1, ALPHA2, ALPHA3, ALPHA4	Penumbra and umbra angles (in degrees) measured from perigee. ALPHA1 is first entry into penumbra; ALPHA2 is exit from penumbra and entry into umbra; ALPHA3 is	(20X, 4F10.0)

Card No.	Variables	Interpretation	Format
		exit from umbra and entry into post-umbra penumbra; and ALPHA4 is exit from penumbra. The angles must be presented in this order, even though ALPHA4 may be numerically smaller than ALPHA1. In the case where only umbra angles, ALPHA2 and ALPHA3, are given, dummy in ALPHA1 and ALPHA4 by respectively subtracting and adding one-half degree to ALPHA2 and ALPHA3. In the case of no eclipse, all four angles are zero.	

Each set of "type a" cards consists of four cards, and these sets or "decks" are arranged serially. The last set of "type a" cards completes the data required for the program.

The data contained on cards 5 through 19 are as follows:

1. For all booms (variables beginning with "B"):

   Field 1: X-centroid coordinate (unshaded)
   Field 2: Y-centroid coordinate (unshaded)
   Field 3: Z-centroid coordinate (unshaded)
   Field 4: Projected area (unshaded)
   Field 5: Length of boom (unshaded)
   Field 6: Blank

2. For all spheres, boxes, cylinders:

   Field 1: X-centroid coordinate (unshaded)
   Field 2: Y-centroid coordinate (unshaded)
   Field 3: Z-centroid coordinate (unshaded)

Field 4:	Projected area (unshaded)
Field 5:	Diameter of sphere (unshaded)
Field 6:	Distance from the body to nearest point of the sphere.

(The torus diameter is considered to be the distance across the loop.)

The options NOSHAD, NEGO, IAIR, ISUN, IGRAV, and ITØRTA are to be used as follows:

NOSHAD	=	0 or 1	depending upon whether effects of shadowing are to be considered or not, respectively. For EGO runs it is more consistent with program logic not to consider shading.
NEGO	=	0 or 1	if the satellite has a torus in the xy or yz plane, respectively.
IAIR	=	0 or 1	Set = 1 if it is desired to skip the effects of aerodynamic torque.
ISUN	=	0 or 1	As in IAIR except concerning solar torque.
IGRAV	=	0 or 1	As in IAIR except concerning gravity-gradient.
ITØRTA	=	1 or 2	If ITØRTA = 1, the intervals at which the torques are computed in an orbit are a function of time. This option is intended for use with the POGO satellite because of its near-circular orbit. If ITØRTA = 2, the intervals are computed as a function of the true anomaly angle (i.e., angle from perigee). The 2 option is intended for use with the near-parabolic EGO orbits.

3. COMPUTATIONS FOR GRAVITY-GRADIENT TORQUES*

$$\omega_o = \sqrt{\frac{\mu}{a^3(1-e^2)^3}}\,(1 + e \cos \nu)^2$$

$$G_x = \frac{4}{2}\,\omega_o^2\,(I_p - I_y)\sin 2\phi \approx 4\,\omega_o^2\,(I_p - I_y)\,\phi$$

$$G_y = \frac{3}{2}\,\omega_o^2\,(I_r - I_y)\sin 2\theta \approx 3\,\omega_o^2\,(I_r - I_y)\,\theta$$

$$G_z = \frac{1}{2}\,\omega_o^2\,(I_p - I_r)\sin 2\,(\psi_g)$$

$$I_{xx} = 660.5 \text{ slug ft.}^2$$

$$I_{yy} = 364.9 \text{ slug ft.}^2$$

$$I_{zz} = 924.8 \text{ slug ft.}^2$$

$$\phi = -0.57^\circ \pm 0.4^\circ$$

$$\theta = -0.08^\circ \pm 0.4^\circ$$

$$\psi_g = -0.57^\circ \pm 1.0^\circ + \psi^\circ.$$

The plus or minus signs are determined by the prevailing torque created by solar radiation and aerodynamic forces.

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* The program has now been amended so that the moments of inertia and principal angles are read in as data. That is, I_{xx}, I_{yy}, I_{zz}, and ϕ, θ, ψ_g are not constants as given above, but are variables that may be changed to suit various configurations of the spacecraft. The equations for G_x, G_y, and G_z remain as given above.

Gravity-gradient computations employ equations which take account of the gyroscopic effects due to the rotation of the satellite at orbital rate. Gravity-gradient torques have a semidependence on roll, pitch, and yaw angles. In the case of roll and pitch, angles are nominally zero, due to attitude control. But small angular deviations arise from two sources:

- Error angles associated with the control system
- Bias angles due to the slight displacement of the principal coordinate system from the body-centered system of the spacecraft.

Since cross-product moments of inertia were very small, they were neglected in the gravity-gradient equations.

When actual values for gravity-gradients were developed from these equations, it was discovered that most gravity-gradient experienced is due to the yaw angle. This is an important observation since, neglecting product inertial terms, yaw gravity-gradient torque vanishes in the absence of gyroscopic effects. In other words, had the satellite been inertial rather than rotating at orbital rate, overall gravity-gradient torques would be greatly reduced.

The importance of developing this gyroscopic component in the yaw gravity-gradient equation makes an appreciable difference to the total POGO gas budget, as is shown later.

4. SOLAR RADIATION DEGRADATION FACTOR

When in penumbra, the satellite encounters less radiation from the sun. The visible area of the sun's disc is computed by assuming the earth to be a straightedge moving across the face of the sun. Then the radiation constant is degraded by the factor of the fraction of the total area that is visible. In the orbits that have been used to date, very little time (1° per orbit) is assumed to be spent in penumbra. This portion of the program should assume more importance in the case of near-parabolic orbits.

5. OPTIONAL PRINTOUT FOR EACH INTERVAL

To obtain this printout, let the print option be 1 on card 1a. The program gives:

Orbit variables for time interval η (heading)

Time in minutes and seconds (time into orbit from perigee)

Mean anomaly in degrees and radians

Eccentric anomaly in degrees and radians

True anomaly in degrees and radians

r, h (height above earth's surface), velocity in ft./sec., radial component of velocity, perpendicular component of velocity

γ, η

α

See definitions of orbit parameters for these terms.

ψ

See definitions of orbit parameters for these terms.

ϕ_p

Gravity-gradient torque (3 components) in body coordinate system

Aerodynamic torque (3 components) " " " "

Solar radiation torque (3 components) " " " "

Gravity-gradient torque (3 components) in inertial coordinate system

Aerodynamic torque (3 components) " " " "

Solar radiation torque (3 components) " " " "

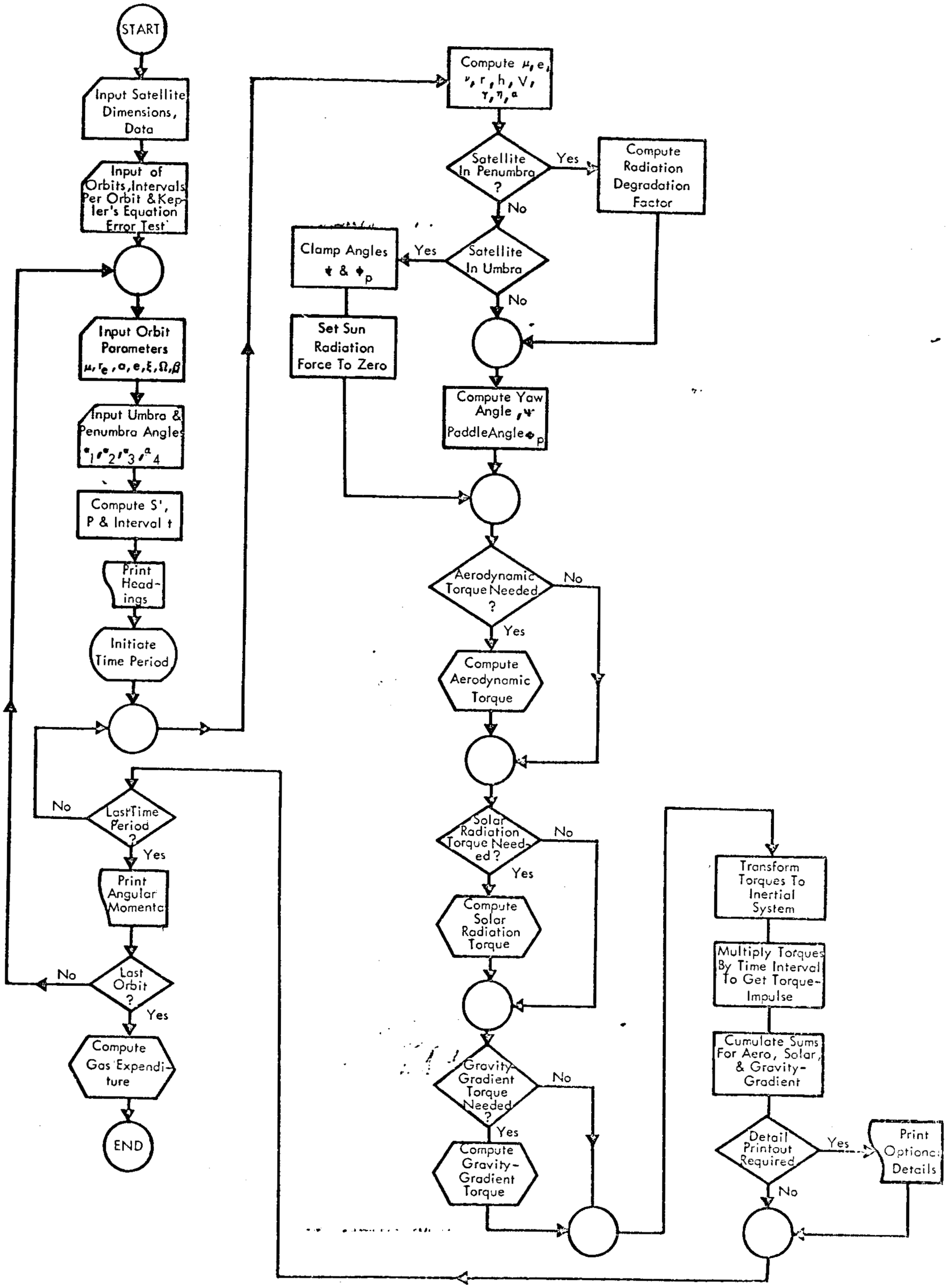
Δt, time interval change, by which the torques are multiplied to obtain torque impulse

Sum of torque impulses about the orbit to this time, in nine components, three each for gravity-gradient, aerodynamic, and solar radiation

Sum of x, y, and z torque impulses from gravity-gradient, aerodynamic, and solar radiation individual components (presented as XSUM, YSUM, and ZSUM)

Total gas for the orbit in pound-seconds

Total gas for number of days elapsed (includes gas for previous orbits). Thus, after the last orbit, the total gas (in pound-seconds) represents gas used in all orbits for total days aloft.



FLOW CHART OF PROGRAM

II. AERODYNAMIC AND SOLAR TORQUE SUBROUTINES

II. AERODYNAMIC AND SOLAR TORQUE SUBROUTINES

The classic equations for aerodynamic drag in an atmosphere that conforms to the restrictions of hyperthermal free molecular flow were used. In simplest form this is:

$$F = \frac{1}{2} \rho v^2 \cdot A \cdot Cd ,$$

where:

ρ	=	density of air
v	=	velocity
A	=	unshaded projected area
Cd	=	drag coefficient.

For a flat plate this equation is decomposed as:

$$F_n = 2qA\,(2 - \sigma')\,\sin^2\theta \quad \text{(normal force)}$$

$$F_t = 2qA\,\sigma\,\sin\theta \cdot \cos\theta \quad \text{(tangential force)}$$

where:

q	=	dynamic pressure $= \frac{1}{2} \rho v^2$
θ	=	angle of attack measured between the plane of the surface and the velocity vector

σ' = normal momentum exchange coefficient

σ = tangential momentum exchange coefficient .

corresponding to those found in STL report GM-61-9721, 4-18.

The equations derived in the aforementioned report were used for the appropriate members of the body, with some adjustment of the signs of the forces. For the appendages--such as the booms, the torus for experiment five, the SOEP antenna, the sphere of experiment six, and the OPEP supporting cylinder--formulas were derived to take into account the extra reflective factors due to curvature. The area of the horizontal cylinders (such as the booms) is still dependent upon the angle of attack; and the sine, cosine relationship used with the flat plate is assumed. The area of the vertical cylinder of OPEP is independent of the yaw angle, so the trigonometric functions enter only once, in the decomposition of the force.

For all parts (including the body) except the torus, the flight path angle, γ, was considered to be zero. This assumption does not affect the results to any appreciable degree because of the inverse relationship between flight path angle and distance from perigee. However, the flight path angle is important for the torus. With an angle of zero, the projected area is a rectangle; whereas with an angle of 90 degrees, the corresponding area is that of a ring. A two arc function was used to approximate this area change. A linear arc carries the projected area from a rectangle to twice that as the flight path angle varies, unshadowing the back portion of the torus. From then it varies as a sine function until at 90 degrees the area is equal to π times that at zero degrees. Small flight path angles are important since a study of the geometry shows that a value of $\gamma = 1.5^{\circ}$ is sufficient to completely unshadow the back half of the torus.

The above treatment assumes that the EP5 torus is imbedded in the xy plane, as it is for the EGO satellite. For POGO satellites, where it is imbedded in the yz plane, a different but analogous handling of the torque-dependence upon spacecraft orientation is used. Selection between the two alternatives is automatically controlled by an input marker signifying whether the run is to be under EGO or POGO conditions.

Since it was decided to ignore interbody shadowing effects, the auxiliary antennae were ignored. In some cases these short boom-like structures would almost totally shadow each other, depending upon slight variations of flight path angle. With no shadowing, the torques produced by them were small and very nearly self-canceling. Torques for all the other small objects may be computed through proper read-in of data; they will be very small.

For the aerodynamic shadowing the results of STL report GM-61-9721, 49, were used for the main body. The shaded area of the booms and the corresponding change of centroid were derived. For all other objects, if a shadow fell across more than one-half the projected area, it was considered totally shaded; if less than one-half, the whole area was used. No shadowing was computed for the solar torques since the only source of shadow is the body itself, so this is negligible.

Some examples of the forms of the equations used will be given. Only the aerodynamic are given, since if one considers the equations for solar forces on a flat plate:

$$F_n = V \cdot A \cdot \sin^2 \theta \cdot (1 + a_s)$$

$$F_t = V \cdot A \cdot \sin \theta \cdot \cos \theta \cdot (1 - a_s)$$

where:

V = pressure constant

A = unshaded area

θ = angle of incidence

a_s = reflectivity of surface

and for notational convenience, let

$$a_s = 1 - \sigma'$$

$$(1 - a_s) = \sigma$$

$$F = V \; ;$$

then the equation will be directly applicable for solar.

For aerodynamic, consider

$$F = \rho v^2 = 2q$$

$$\theta = \text{angle of incidence} \; ;$$

then the forces will be approximately as follows:

(1) Flat Plate

$$F_n = F \cdot A \cdot \sin^2 \theta \left\{1 + (1 - \sigma')\right\}$$

$$F_t = F \cdot A \cdot \sin \left|\theta\right| \cos \theta \; \sigma$$

(2) Booms

$$F_n = F \cdot A \cdot \sin^2 \theta \left\{1 + \frac{1}{3}(1 - \sigma')\right\}$$

$$F_t = F \cdot A \cdot \cos \theta \; \sin \left|\theta\right| \left\{1 - \frac{1}{3}(1 - \sigma')\right\}$$

(3) OPEP Cylinder (and high-gain antenna)

$$\text{Force} = F \cdot A \cdot \left\{1 + \frac{1}{3}(1 - \sigma')\right\}$$

$$F_x = F \cdot A \cdot \sin \left|\theta\right| \left\{1 + \frac{1}{3}(1 - \sigma')\right\}$$

$$F_y = F \cdot A \cdot \cos \theta \left\{1 + \frac{1}{3}(1 - \sigma')\right\}$$

(4) Spherical

$$\text{Force} = F \cdot A$$

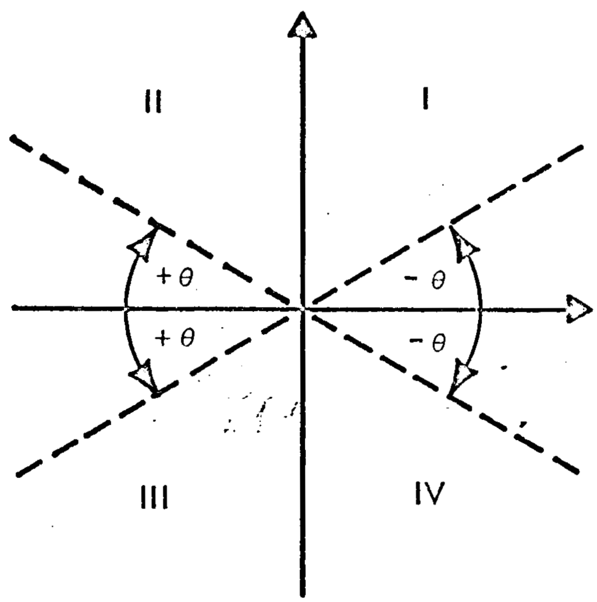
$$F_x = F \cdot A \cdot \sin|\theta|$$

$$F_y = F \cdot A \cdot \cos\theta$$

(5) Torus

$$\text{Force} = F \cdot A \cdot \left\{1 - \frac{1}{9}(1 - \sigma')\right\}$$

The angle θ used in these equations is an adjusted angle according to the scheme diagrammed below:



III. OBTAINING GAS EXPENDITURE, GIVEN THE SECULAR ANGULAR MOMENTUM

III. OBTAINING GAS EXPENDITURE, GIVEN THE SECULAR ANGULAR MOMENTUM

The computer program accepts sets of orbital parameters and accumulates angular momentum over one complete orbit per parameter set.

Angular momentum is output separately for aerodynamic, solar, and gravity-gradient torques, each in the x, y, and z coordinates. As the increments of angular momentum are developed, they are dumped into an inertial system which is conveniently located in the orbital plane, at that point in the orbit lying in the projection of the sun vector. The z-axis passes through the center of the earth, the x-axis is normal to it in the orbital plane, and the y-axis is normal to the orbital plane.

This location of the inertial system was chosen because the yaw angle, ψ, will always be zero at this point. This simplifies the subsequent partitioning of angular momentum unloading between the roll and pitch gas jets. This coordinate system is considered inertial because the orbit plane is held steady during a single revolution.

Between successive sampling points, the orbital parameters are changed to allow for:

- Precession of perigee
- Recession of line of nodes
- Movement of sun vector
- Change in eclipse angles
- Change in perigee height and orbital eccentricity.

A sampling period of 15 days was selected since this corresponds to a 60-degree shift in the argument of perigee. Hence, each set of six successive orbital samples steps the argument of perigee completely around the orbital. *

Given the angular momenta output from the computer, the gas expenditure is obtained by means of the following two successive steps:

- Computation of how the momentum unloading will be shared between the pitch and roll gas jets. This leads immediately to an estimate of gas thrust (in pound-seconds) required to unload the angular momentum for a single revolution

- Multiplication of this expenditure by the number of orbits taking place during the sampling increment (in this case, 15 days).

The computation of gas expenditure from angular momenta described in succeeding pages below was originally done on a desk computer. Subsequently, an addition was made to the main computer

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* Perigee precession and nodal regression rates are assumed constant.

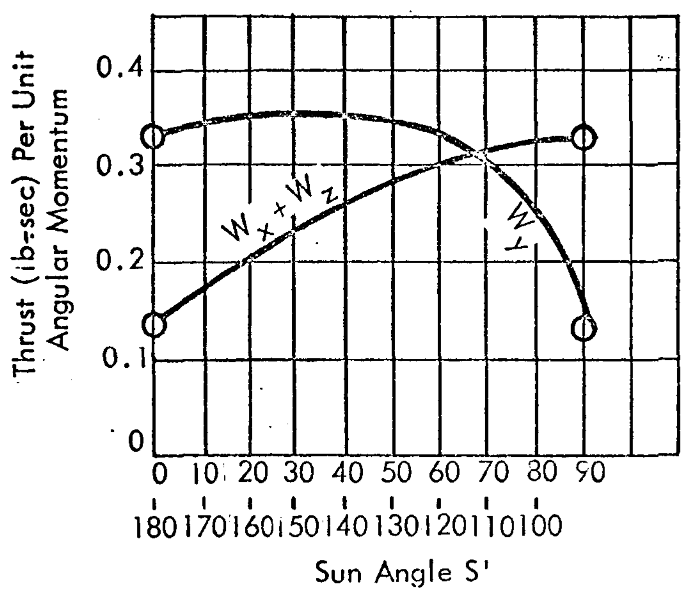
program so that gas budget is now computed automatically and the cumulative value output along with the angular momenta for each orbital. An algorithm identical to that described below is used by the computer.

Since the roll jets have a lever arm of 7.68 feet, then a thrust of $\frac{1}{7.68} = 0.130$ pound-second will be required to unload unit angular momentum (1 pound-foot-second). Similarly, the pitch jets have a lever arm of 3.08 feet and hence will require a thrust of $\frac{1}{3.08} = 0.324$ pound-second to unload unit angular momentum.

How much of the x, y, and z angular momenta must be unloaded by each of the pitch and roll jets depends upon the relation of the orbital to the sun vector. Two special cases clearly have simple solutions:

- When the sun is normal to the orbital plane ($S' = 0^{\circ}, 180^{\circ}$), yaw angle, ψ, is always zero. Hence, all the x and z momenta must be unloaded through the roll jets and the y momentum through the pitch jets.
- When the sun vector lies in the orbital plane ($S' = 90^{\circ}$), ψ is always $\pm 90^{\circ}$ (except for the brief yaw reversal maneuvers at midday and midnight). Hence, all x and z momenta must be unloaded through the pitch jets and the y momentum through the roll jets.

For all other sun angles, all three momenta (x, y, and z) will be partitioned between roll and pitch jets. The effect of this partitioning is to increase the gas expenditure somewhat, since, in effect, momentum is being unloaded by a pair of oblique thrusts instead of by one perpendicular thrust. The amount of gas required to unload unit angular momentum as a function of sun angle is shown in the following graph.



The W_x curve applies to both x and z momenta and the W_y curve to the y momentum. No distinction is made between the x and z angular momenta since both are in the orbital plane and both are passed back and forth between the same mix of pitch and roll inertia wheels (though there is a 90-degree phase difference between them). Absolute values of x and z momenta were therefore added together. Inspection of this curve shows the following:

- At $S' = 0^{\circ}$, 180°, x and z require 0.130 pound-second, corresponding to roll jets only; y requires 0.324 pound-second, corresponding to pitch jets only

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- This is reversed when $S' = 90^{\circ}$.

- At intermediate values, there is a mix between these two extremes. The extent to which both curves are concave downwards (i.e., the extent to which they depart from straight lines) is an index of how much gas is being lost through oblique unloading of momenta.

The above curves were obtained from a consideration of the yaw angle function:

$$\tan \psi = -\tan S' \sin \theta ,$$

where θ is the orbital angle from the sun vector.

The curves were obtained by averaging out $\cos \psi$ and $\sin \psi$ over the orbit, using the above function; this was done by an approximate graphical procedure.

A more precise evaluation would produce a set of curves, rather than a single curve, giving weights as a function of argument of perigee. All these curves would be anchored at the same pair of coordinates at $S' = 0^{\circ}$ and $S' = 90^{\circ}$, and would have the same general shape. This refinement was not attempted since it would be largely "washed out" by uncertainties in the value of the true yaw angle during eclipse. Furthermore, the argument of perigee occupies all angular positions about equally often, so that much of the discrepancy should average out.

To obtain the thrust required to unload the angular momenta accumulated during a single orbit, W_x and W_y were first read off from the above pair of curves, given the sun angle S'. W_x was multiplied by the arithmetic sum of the x and z angular momenta, and W_y by the y angular momentum. These two were then summed to give the total thrust required to unload all the secular momenta.

To obtain the gas budget for a 15-day interval, the single orbit budget is multiplied by the number of orbits occurring in 15 days. This number ranged from 224 for a typical POGO orbital to about 6 for EGO.

Table 1 illustrates the development of gas expenditure. The orbital for this example has an initial perigee of 150 n.m.

Table 1

Secular Angular Momentum per Orbit (15-Day Intervals)

Time (Days)	M_x	M_y	M_z	$\lvert M_x \rvert + \lvert M_z \rvert$	W_x	$W_x(\lvert M_x \rvert + \lvert M_y \rvert)$	W_y	$W_y(M_y)$	Total Thrust per Orbit (Pound-Second)
0	1.490	-0.551	0.185	1.675	0.150	0.252	0.330	0.182	0.434
15	0.580	-0.533	1.370	1.950	0.215	0.419	0.345	0.184	0.603
30	0.389	-0.411	0.057	0.446	0.270	0.120	0.350	0.143	0.263
45	- 1.434	-0.400	0.286	1.720	0.310	0.533	0.310	0.124	0.657
60	0.770	-0.247	2.570	3.340	0.324	1.080	0.130	0.032	1.110
75	- 1.120	-0.075	-0.405	1.525	0.310	0.473	0.300	0.023	0.496
90	0.748	-0.360	-0.259	1.007	0.275	0.277	0.345	0.124	0.401
105	1.269	-0.598	-1.377	2.646	0.230	0.608	0.350	0.209	0.817
120	1.249	-0.770	-1.850	3.099	0.225	0.695	0.350	0.269	0.964
135	- 0.443	-0.749	-2.250	2.683	0.260	0.697	0.350	0.262	0.959
150	- 1.250	-0.881	-2.259	3.509	0.295	1.035	0.325	0.286	1.321
165	- 4.345	-1.217	-0.468	4.813	0.320	1.540	0.250	0.317	1.857
180	- 2.950	-0.138	1.527	4.477	0.320	1.430	0.275	0.038	1.468
195	- 0.831	-0.431	-1.510	2.341	0.290	0.678	0.330	0.142	0.820
210	- 2.630	-0.590	-2.350	4.980	0.245	1.220	0.350	0.206	1.426
225	- 3.870	1.010	-2.420	6.290	0.215	1.355	0.348	0.352	1.707
240	- 5.920	-1.176	-3.050	8.970	0.230	2.060	0.350	0.412	2.472
255	- 9.050	-0.886	1.190	10.240	0.275	2.830	0.345	0.305	3.135
270	- 5.260	-0.785	6.050	11.310	0.310	3.510	0.300	0.235	3.745
285	- 2.970	-0.554	-5.180	8.150	0.324	2.640	0.130	0.072	2.712
300	- 9.360	-0.650	-2.070	11.430	0.305	3.480	0.315	0.204	3.684
315	-13.470	-2.650	13.500	26.970	0.265	7.150	0.350	0.927	8.077

IV. COMPUTED GAS BUDGETS FOR POGO AND EGO

IV. COMPUTED GAS BUDGETS FOR POGO AND EGO

1. DESCRIPTION OF FIGURES 1 TO 4

These graphs present the cumulated gas expenditure as a function of time, plotted at 15-day intervals, under various conditions. *

Figure 1 presents a set of POGO curves, four at an initial perigee of 150 n. m. and one at 155 n. m. The four 150-n. m. curves show the effects of different boom configurations upon gas consumption. Reckoned in terms of the number of days required to exhaust 700 pound-seconds of gas, it is seen that:

(1) Disregarding all booms gives a life of about 270 days

(2) Including all booms except EP5 torus and SOEP antenna gives a life of about 180 days

(3) Including all booms gives a life of about 68 days**

(4) When the EP5 torus is rotated 90° into the xy plane (as in EGO), the life goes up slightly to about 78 days.**

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* See Appendix C for orbital parameter histories upon which these runs were based.

** Since these two runs were made, an error has been discovered in the corresponding input data. As a result, the true curves would show a somewhat higher expenditure than those shown in the diagram. These curves were not rerun, since the corresponding gas expenditures will clearly be unacceptable.

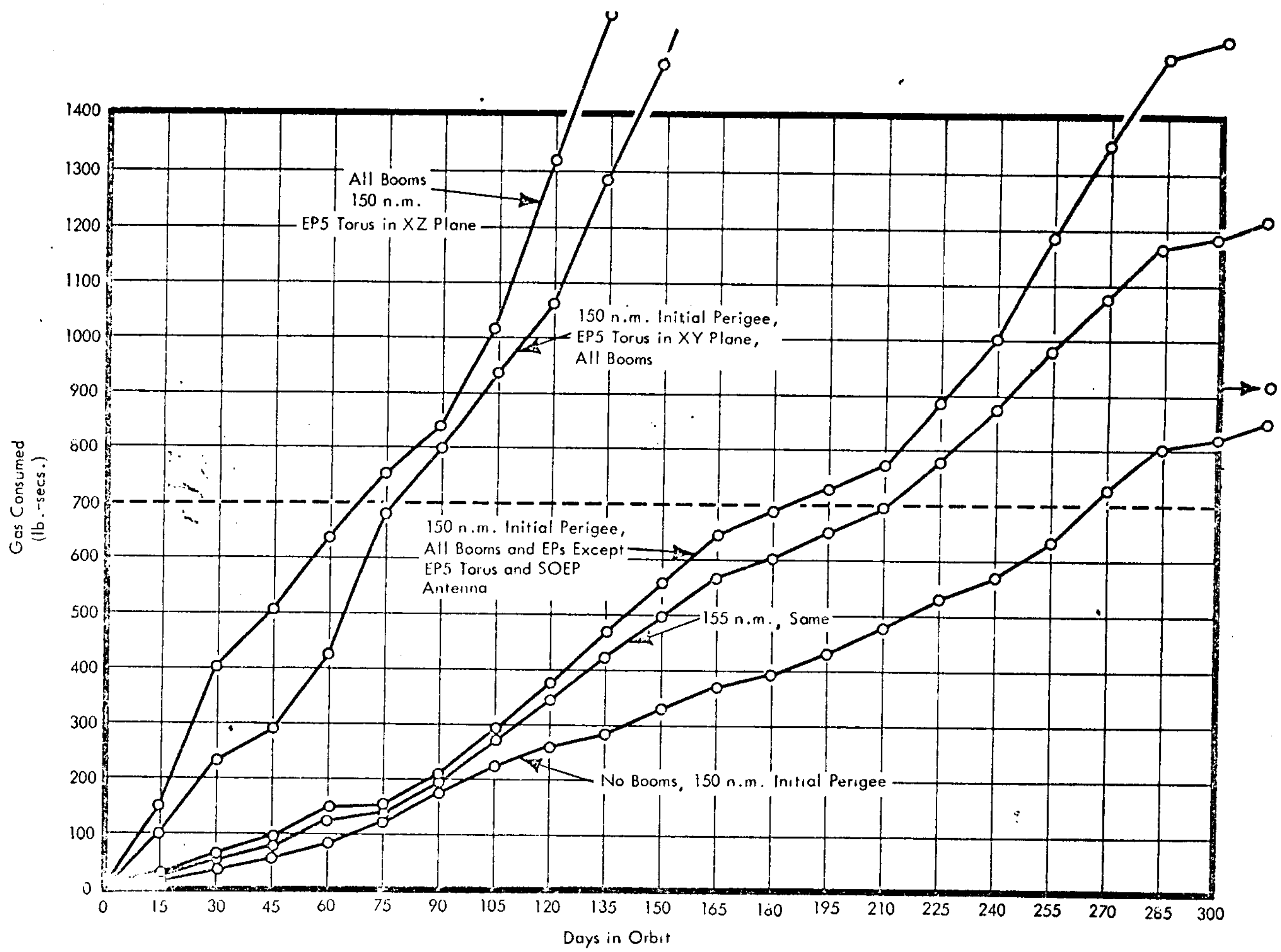

Figure 1 POGO GAS EXPENDITURE

When case (2) above was run at 155 n.m. instead of 150 n.m. perigee, the life was seen to increase from 180 days to about 210 days..

Figure 1 reveals that the presence of booms makes an immense difference to the gas budget expenditure of POGO. It also confirms what had already been forecast from preliminary computations based upon the boom dimensions and parameters--that most of the noncanceling boom torque was due to the EP5 torus and the SOEP antenna.

Most of these, and the curves in the subsequent graphs, show an increase in slope as time progresses, due to the gradual sinking of perigee caused by atmospheric drag. Many of the curves also show a "humping" with an approximately 90-day period, due to the cyclic effects of solar perturbations of perigee height and the effects of the corresponding changes in the inclination of the sun to the orbital plane.

Figure 2 explores the effect upon gas expenditure of withholding the deployment of the EP5 torus or the antenna. Simulations were run at both 180 n.m. and 200 n.m. initial perigee. Assuming a 700-pound-second gas budget, lifetimes under the various conditions are seen to be as follows:

(1) No SOEP antenna, but with EP5 torus deployed, gave a lifetime of about 120 days at 180 n.m. initial perigee

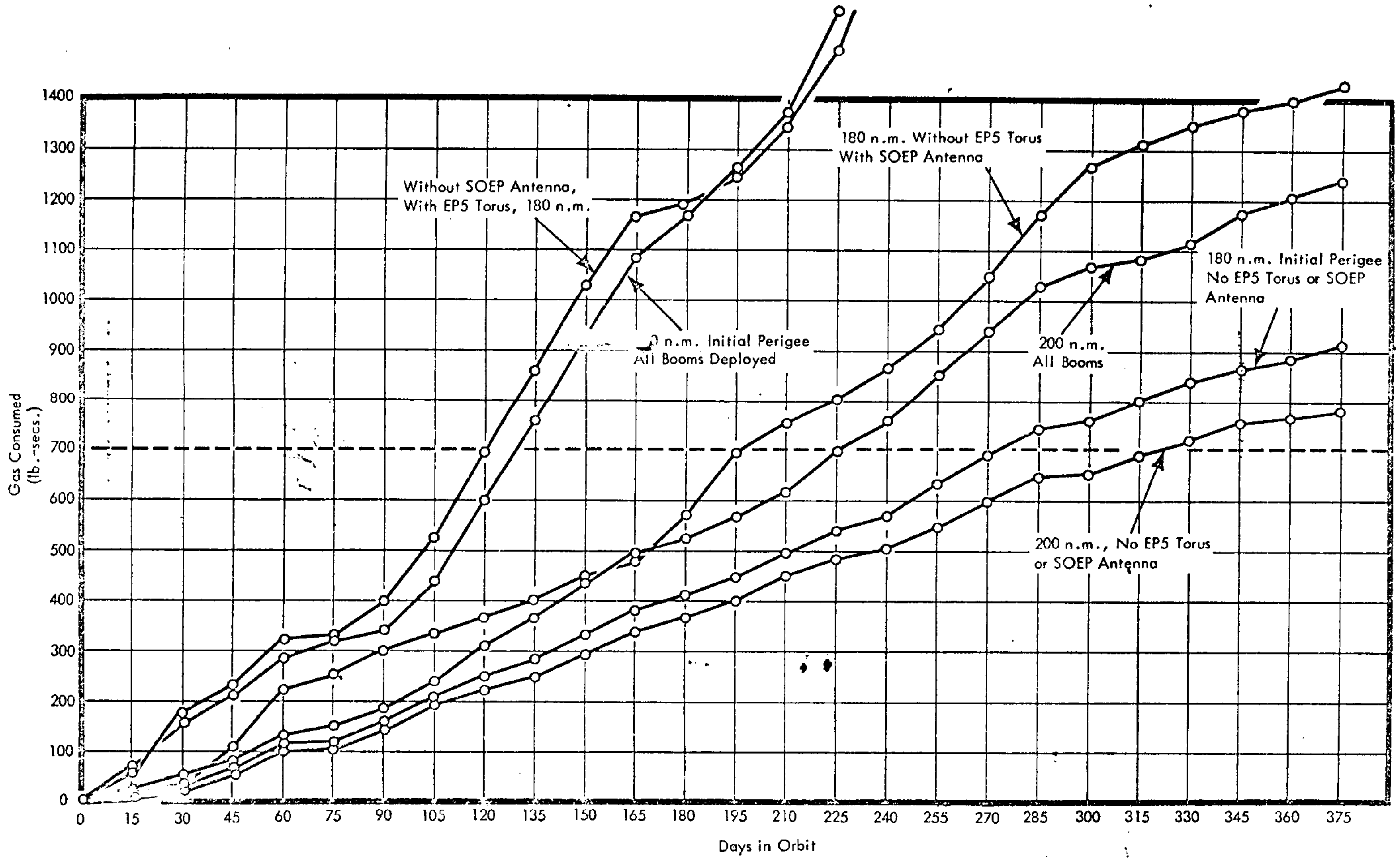

Figure 2 POGO GAS EXPENDITURE

(2) No EP5 torus, but with the SOEP antenna deployed, gave a lifetime of about 195 days at 180 n. m. initial perigee

(3) Neither EP5 torus nor SOEP antenna deployed gave a lifetime of about 275 days at 180 n. m. initial perigee

(4) The corresponding lifetime at 200 n. m. rose to about 322 days

(5) All booms deployed gave a lifetime of about 130 days at an initial perigee of 180 n. m.

(6) The corresponding lifetime at 200 n. m. initial perigee rose to about 225 days.

The shape of the curve corresponding to case (1) above was totally unexpected; it shows that in the presence of the EP5 torus, withholding deployment of the SOEP antenna may <u>decrease</u> the lifetime. This directly contradicts the effects of withholding its deployment in the absence of the EP5 torus, when a gain of 65 days' life was obtained. The explanation lies in the complex boom geometry and the resulting dependence of torque on yaw angle ψ. The SOEP antenna exerts maximum torque when $\psi = 90^{\circ}$. At $\psi = 90^{\circ}$, the EP5 torus generates an opposing torque due to the offset in its boom along the x-dimension. But in the absence of the EP5 torus, the large SOEP antenna torque is mainly unopposed, hence exerting an appreciable effect upon the gas expenditure.

Two special curves are shown in Figure 3. The first gives the gas expenditure history for 180 n.m. perigee with all booms deployed but with all aerodynamic torques suppressed. The intention was to obtain a limiting, or benchmark, curve showing the maximum possible increase in lifetime obtainable by raising the perigee to gain the benefit of a thinner atmosphere. As is seen, under these limiting conditions, the lifetime is still slightly short of one year.

The second curve attempts to obtain a gas budget history under conditions as closely resembling that made previously by STL. The details are as follows:

- All booms were excluded
- OPEP was included, but the supporting cylinder was excluded
- Paddle x Box shadowing was included
- The run was made at an initial perigee of 180 n.m.
- Gravity-gradient yaw torques due to gyroscopic effects caused by the orbital-period rotation of the satellite were excluded. This was done because Otten's report* gives equations for gravity-gradient computations which appear to exclude gyroscope effects.

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* D. D. Otten, "OGO Attitude Control Subsystem Description, Logic, and Specifications," Space Technology Laboratories, Inc., 2313-0004-RU-000, December 1961, p. B 10.

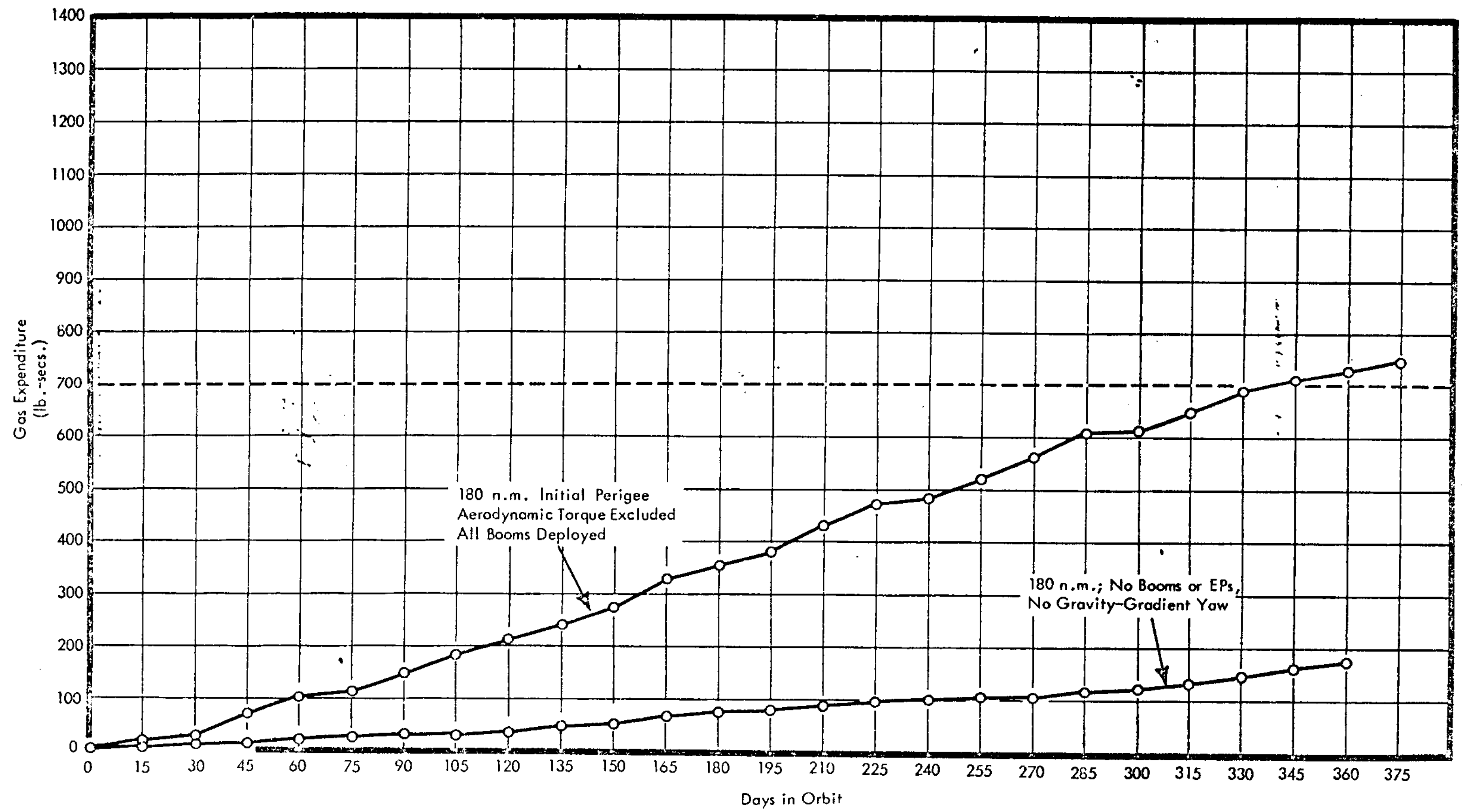

Figure 3 POGO GAS EXPENDITURE

We obtained a gas budget expenditure of about 154 pound-seconds. This is about half STL's value. However, other differences between the two simulations have since come to light. Specifically, it appears that STL used the ARDC 1959 standard atmosphere, which is more dense at orbital altitudes than the "quiet sun" atmosphere we used (S = 75). But tending to offset this is the fact that STL used a perigee altitude of 200 n. m. In addition, we are unsure of the exact way in which STL handled the development of OPEP torques. In view of all this, it is believed that the two programs cross-check as closely as could be expected.

Figure 4 presents the gas budget history for EGO. As expected, the total expenditure was far less than for POGO, owing to the small fraction of the orbital period spent in the near-earth environment.

2. EXAMINATION OF SOME BIASES AND ERRORS

Biases and errors inherent in the gas budget computations presented above are listed in the following table, where a distinction is made between program defects and uncertainties in the values of inputs accepted by the program.

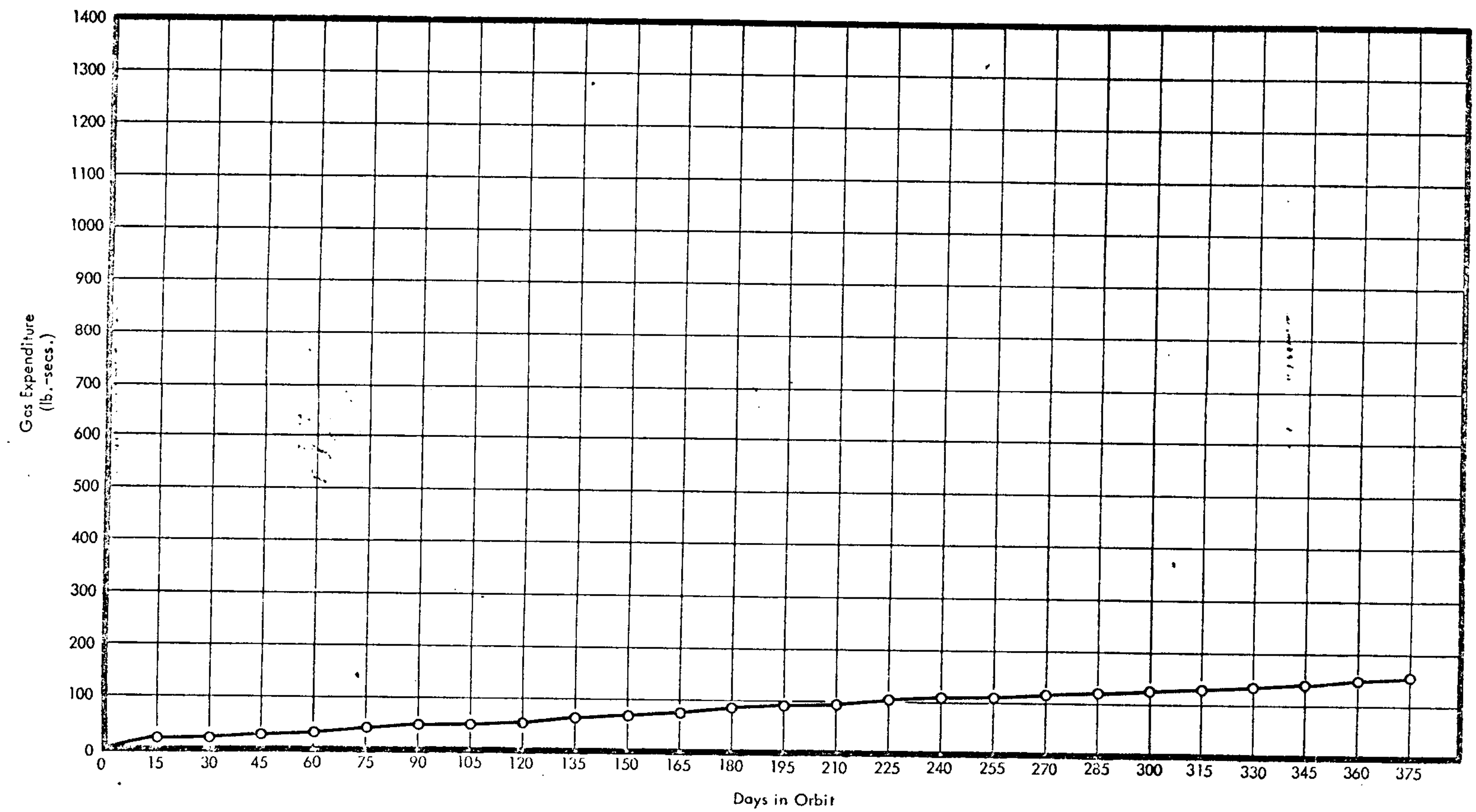

Figure 4 EGO GAS EXPENDITURE

	Major Sources of Error and Bias	Minor Sources of Error and Bias
Program Input Errors	(1) Uncertainties in atmospheric density (2) Uncertainty in value of aerodynamic reflection coefficients	(1) Orbital parameter sampling errors
Program Defects	(3) Unknown yaw angle history during eclipse (4) Omission of coulomb drag effects	(2) Earth's radiation torques omitted (3) Induced electromagnetic torques omitted (4) Effects of micrometeoroids and solar wind omitted (5) Partition of yaw momentum unloading between pitch and roll jets is inexact

Atmospheric density estimates vary according to which atmospheric model is used and the solar activity. Our gas budget computations used an atmospheric height/density profile obtained from the publication "The Upper Atmosphere in the Range from 120 to 800 Km," issued by the Institute for Space Studies. We selected their "quiet sun" model (S = 70). We feel that these density values may certainly be in error by a factor of two.

The aerodynamic reflection coefficient σ' cannot be determined experimentally (due to the impossibility of developing the hard vacuum required), and its theoretical derivation inevitably rests on unverifiable assumptions concerning the thermodynamic interaction between impinging air molecules and the spacecraft surface. This is examined in more detail in the next section. In consequence, an uncertainty which may easily amount to $\pm$ 50 percent is introduced into the aerodynamic torque computation.

Probably the greatest defect in the program itself concerns the treatment of yaw angle during eclipse. In the OGO spacecraft, control of the yaw angle is interrupted as soon as solar lockon is lost. The yaw inertia wheel is then allowed to run down, transferring its angular momentum to the spacecraft as it does so. In order to develop a reasonably accurate yaw angle history during an eclipse, the following factors would have to be included:

(1) Run-down time function of the yaw inertia wheel

(2) Yaw angle rate at the moment of entering eclipse

(3) Torque history during the eclipse period.

Of these three factors, the present program develops only the last. Factor (2) presents the greatest difficulty, since our program assumes perfect attitude control of the spacecraft and does not carry angle rate information. The convention was therefore adopted of holding the yaw angle constant during the eclipse period. Although the size of the error so introduced cannot be computed, a straw-in-the-wind is provided by a comparison of the gas consumption of certain "standard" orbits computed with and without loss of yaw angle control. A difference of up to 30 percent was observed for some orbit orientations; though for others, especially those with shorter or zero eclipse periods, the difference observed was little or nothing. These data are expanded below in "Computer Parametric Study."

Since the "atmosphere" which OGO is moving through most of the time is really a plasma, and also because of the photoelectric effect, the spacecraft acquires an electric charge. This in turn entrains a film of plasma which effectively increases the projected area of cross section of all parts of the spacecraft, hence increasing atmospheric drag. This increased area of cross section can be roughly computed using the hypothetical "debye length," which is computed as follows:*

$$h = \text{debye length} = \sqrt{\frac{\kappa T}{4 \pi \eta_e e^2}} = 6.9 \left(\frac{T}{\eta_e}\right)^{\frac{1}{2}}$$

where:

κ = Boltzmann constant = 1.380×10^{-16} erg/degree

e = Charge on proton = 4.803×10^{-10} ESU

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* See Lyman Spitzer, Jr., Physics of Fully Ionized Gases, Wiley, 1962.

T = Temperature, ^{o}Kelvin

η_e = Number of electrons per cubic centimeter.

Thus, for example, if $T \simeq 2000^{o}C$ and $\eta_e \sim 10^5/cc$, the corresponding debye length is 1 cm. The extension of the projected area of a compact satellite like Vanguard, 1 cm in all directions makes no appreciable difference to drag. But satellites like OGO are appreciably affected, due to a very large perimeter created by the various booms and appendages. Debye lengths of one-half inch to one inch are possible in the thicker parts of the atmospheric plasma through which POGO moves, causing a corresponding drag increase of 14 to 30 percent.

The minor sources of error will now be briefly commented upon. First is that arising from the sampling of orbital parameter values. The present program computes gas expenditures for a succession of single orbits which are spaced throughout the year's lifetime of the satellite. Orbital parameters are assumed to hold constant during a single revolution. Each such orbit has a separate set of input parameters, these being adjusted to allow for orbital changes occurring in the elapsed time at which successive orbits are taken. This sampling procedure saves computation time (e.g., in the POGO gas budget computation, the total number of orbits amounted to 5400, from which 24 were sampled for gas budget computations). It also avoids the need for orbital perturbation subroutines which would have considerably enlarged the program. This is achieved at the cost of introducing sampling error. This may be held to a minimum by application of good sampling practice, i. e., by ensuring that orbitals are sampled representatively with respect to those factors influencing rate of gas consumption. Three such factors are: perigee height as a function of time, inclination of the orbital plane to the sun, and value of the argument of perigee.

Earth's radiation (and reflected solar radiation) was ignored. Although these two factors combined may sometimes approach direct solar radiation, this should not perturb gas budget computations seriously, since:

- In POGO, solar torques are far outweighed by aerodynamic and gravity-gradient torques
- In EGO, the satellite spends only three percent of its time within one earth radius of the earth.

Torques from electromagnetic interactions with the earth's magnetic field were found to be trivial since the means of developing electric current circuits of sufficient magnitude within the spacecraft did not appear to exist. By no stretching of the imagination could we develop an electromagnetic yaw torque which approached that caused by gravity-gradient closer than about two orders of magnitude.

Micrometeoroids and solar winds were disregarded since they appeared to develop forces two or more orders of magnitude less than the major forces considered.

Since all yaw angular momentum must be unloaded by the pitch and roll gas jets, the amount of gas required to do this will be a function of the altitude of the spacecraft as each increment of torque is acquired. To compute the gas expenditure accurately would require a dynamic program which carried reaction wheel loadings at all times so that individual gas firings could be simulated. This was not possible in our nondynamic program. Hence, a statistical averaging procedure was used (see Section III). It does not appear that the error entailed should be more than a few percent at most.

In summary, it appears that the gas budget estimate of POGO is beset by many errors which together amount to something in the neighborhood of a fourfold error, insofar as this can be estimated. EGO gas budget estimates, however, should be quite accurate. This is because the small aerodynamic torque impulse per orbit renders uncertainties in this torque innocuous, and because eclipses occupy at most only a small fraction of the orbital period. Hence, yaw angle is usually controlled, and all the factors required to compute gravity-gradient and solar torque impulses are subject to only small errors.

3. DISCUSSION OF REMEDIES

From the curves presented earlier in this section, it is evident that no amount of raising of the initial perigee (within reason) will give a year's life, given an available gas budget of 700 pound-seconds.

If all booms are deployed throughout, it appears that about a half-year's life would be obtained, given an initial perigee of 190 n. m.

If the EP5 torus and the SOEP VLF antenna are both undeployed, a lifetime of about 275 days should be obtained at 180 n. m. initial perigee. If the decision is made to deploy these appendages at some epoch t during the year, then the corresponding gas budget may be obtained simply by lowering the "all booms deployed" curve till it intersects the "no EP5 torus or SOEP VLF antenna" curve at epoch t and then reading off the date at which 700 pound-seconds of gas (or other value) are expended. If telemetered housekeeping data giving control-gas pressure are available for any epoch t after launch date, then this information can be used to adjust the slope of the gas budget expenditure curve obtained by simulation. This will give a refined estimate of the expected lifetime and hence will provide a more solid basis for ground-control decisions, e.g., for deploying previously undeployed booms.

We understand that one method of gas expenditure reduction currently under consideration is the addition of a compensating sale in an attempt to balance out asymmetric pressures which are the cause of the high aerodynamic torques.

In the course of developing the aerodynamic torque equations for the gas budget simulation model, drag coefficients had to be developed for variously shaped components. These drag coefficients are functions of σ and σ'. We early became impressed by the sensitivity of drag coefficient values to those assumed for σ and σ'.

If all drag coefficients were of the same form, then any bias in the values of σ and σ' would merely scale all torques proportionately, including that due to the added sail. But when different types of drag coefficient are simultaneously present, a bias in σ and σ' would upset the compensating effect of the sail, possibly severely.

A simple example will drive this home. The force equations we used were:

$$\left.\begin{aligned} p &= (2-\sigma')\,p_i + \sigma' p \\ \tau &= \tau_i - \tau_r = \sigma\tau_i \end{aligned}\right\} \text{For plate surfaces}$$

where:

σ' = pressure reflection coefficient

σ = tangential stress reflection coefficient

p = pressure due to Maxwellian rebound

p_i = impact pressure

τ_i = incident tangential stress

τ_r = reflected tangential stress.

Consider the first of the above equations. We dropped the second term, since it appears that wall temperatures to be expected in POGO will lead to low-energy Maxwellian rebound.

> ...If the subsequent re-emission is completely diffuse, it will leave associated with it a momentum flux p_{r}, which is orders of magnitude less than the incident flux. *

When $\sigma' = 0$, there is 100 percent specular reflection; when $\sigma' = 1$, reflection is totally Maxwellian.

For convenience, put $1 - \sigma' = r$, so that r corresponds to the proportion of rebound which is specular. Then, $(2 - \sigma')\, p_i$ becomes:

$p_i\,(1 + r)$ for plate surfaces

---

* Evans, Torques and Altitude Sensing in Earth Satellites, edited by Fred Singer, Academic Press, 1964.

or $p_i (1 + rf)$ for surfaces other than plates, where f is a coefficient which is a function of the shape of the surface.

The term in parentheses is equal to half the drag coefficient. Thus, the drag coefficient may be written:

$$C_d = 2(1 + rf).$$

Our model considers four kinds of shapes with the following f values:

Plate: $f = 1$

Cylinder: $f = \frac{1}{3}$

Sphere: $f = 0$

Torus (edge-on)*: $f = \frac{1}{9}$

The following table contrasts drag coefficients for these various shapes as σ' is changed from 0.8 to 0.2:

Shape	Drag Coefficient Formula	Drag Coefficient	
		$\sigma' = 0.8$	$\sigma' = 0.2$
Plate	$2(1 + r)$	2.40	3.60
Cylinder	$2(1 + \frac{1}{3}r)$	2.14	2.53
Sphere	$2(1 + 0)$	2.00	2.00
Torus (edge-on)	$2(1 - \frac{1}{9}r)$	1.96	1.82

* Courtesy of Ben Zimmerman, GSFC.

Hence, when σ' goes from 0.8 to 0.2, the plate drag increases by 50 percent, the cylinder drag by 20 percent, and the sphere drag remains unchanged. The edge-on torus drag moves in the reverse direction, going down slightly. (The EP5 torus antenna is edge-on when yaw angle $\psi = \pm 90$. When $\psi = 0$, torus is perpendicular to the wind, and the drag coefficient corresponds to that of a cylinder.)

Following STL, the values we are currently using for both σ and σ' are 0.8. Other authorities appear to support high values for σ and σ'.

> ...No empirical values of σ' have been obtained at present. It will be noted, however, that for air incident on most surfaces, $\alpha \simeq \sigma \simeq 1$. It is, therefore, to be expected that $\sigma' \simeq 1$ also. *

But conflicting opinions have also been found:

> ...More recently, molecular beam experimentation has indicated a downward shift in these values ....It is entirely possible that all prior dates will have been discredited. The coefficients may be found to vary greatly from one another, as functions of surface, temperature, speed, and incidence. **

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* Handbook of Supersonic Dynamics, Section 16, Mechanics of Rarefied Gases, Navord Report 1488, Volume 5.

** Evans, op. cit.

R. Schausberg, in Rand Report RM-2313 ("A New Analytic Representation of Surface Interaction for Hyperthermal Fall Molecular Flow with Application to Neutral-Particle Drag Estimated of Satellites"), after a consideration of the thermodynamics of the interaction of air molecules with the wall surface, develops an expression for σ' which under POGO conditions would be 0.057.

V. ANALYSIS OF TORQUE ORIGINS AND THEIR DEPENDENCE UPON ORBITAL PARAMETERS

V. ANALYSIS OF TORQUE ORIGINS AND THEIR DEPENDENCE UPON ORBITAL PARAMETERS

1. BREAKDOWN OF AERODYNAMIC TORQUES

In order to obtain some feel for the torque contributions made by various components and appendages of the spacecraft, normalized torques were computed for yaw angles of 0° and 90°, respectively. The breakdown is presented in Figure 5 in bar diagram form. Normalized torque is that torque which would be obtained for unitary ρv^2.

It is clear that when $\psi = 0^\circ$, EP5 and associated torus contributes the greatest single torque. Further, it is seen that in the absence of the EP5 torus, all other torques are largely self-canceling.

Differences between POGO and EGO result largely from the orientation of the torus; in the case of POGO it is in the yz plane, while in EGO it is in the xy plane. As a result, the whole toroidal loop is normally exposed to the wind in POGO ($\psi = 0^\circ$); whereas in EGO, the projected area of the torus depends on flight path angle γ. When $\gamma = 0^\circ$, only the front edge of the torus is exposed to the wind; when $\gamma = 1.5^\circ$, the rear edge of the torus is unshadowed; finally, when $\gamma = 90^\circ$, the

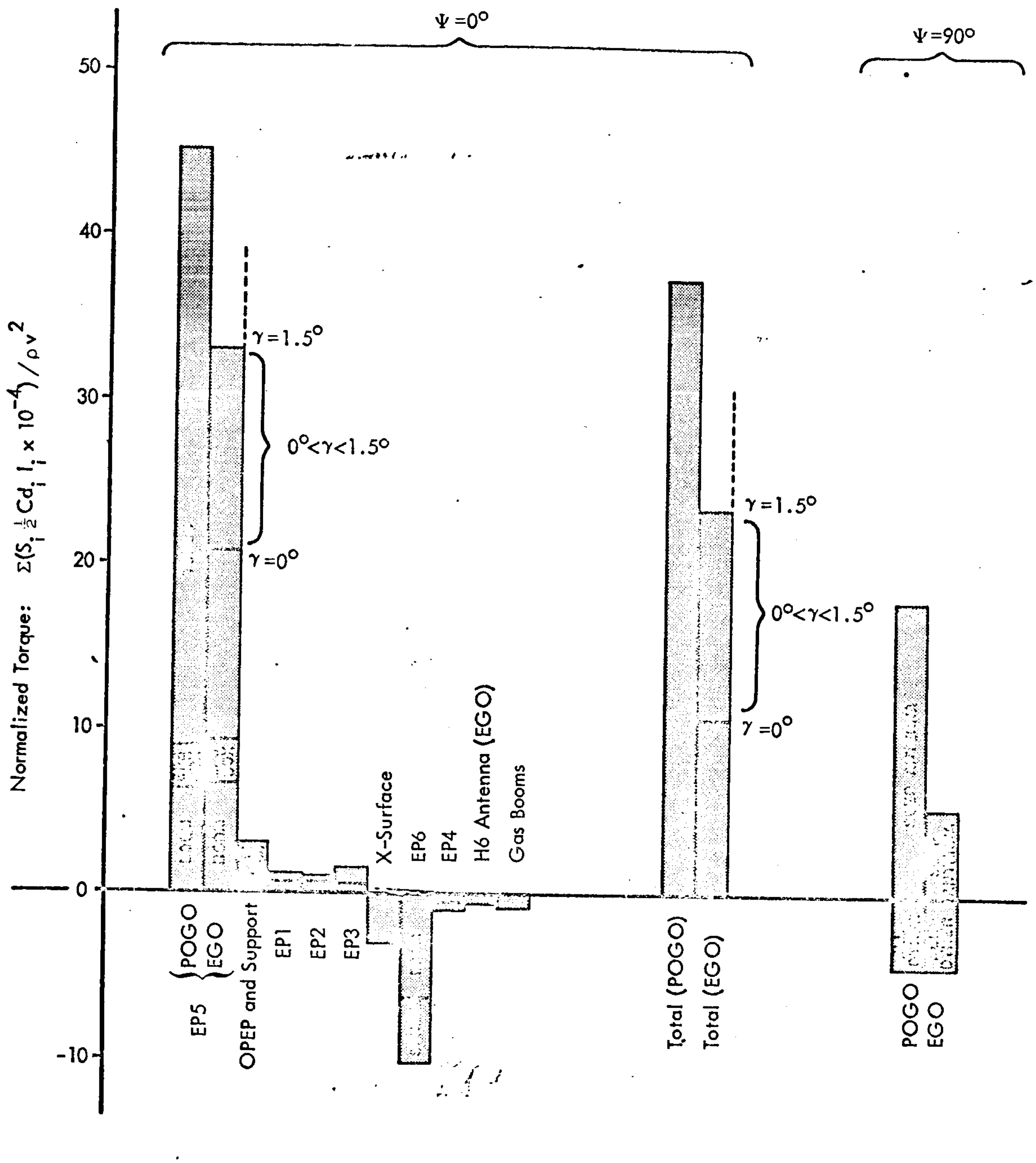


Figure 5 ANALYSIS OF AERODYNAMIC TORQUES

exposure is the same as for POGO at $\gamma = 0^{o}$. (The flight path angle in POGO can be neglected since its maximum value is only 3^{o}.)

When $\psi = 90^{o}$, the SOEP antenna is the major contributer. The torque is greater for POGO because the antenna is 60 feet long, compared to 30 feet for EGO.

Figure 6 shows the effects of debye lengths of one-half inch and one inch upon net torques. Again, differences between POGO and EGO are largely due to differences in torus orientation. In addition, the high-gain antenna in EGO adds about eight percent torque to the total, for debye length of one inch, due to its very large perimeter.

These two bar charts do not give any feel for the dependence of aerodynamic torques upon yaw angle since this is swept through 360 degrees. The relationship is a complex one, due to the subtleties of (Box) x (Paddle) and (Paddle) x (Boom and EP) shadowing. Figures 7 and 8 present aerodynamic torques as a function of yaw angle. Of particular interest is the asymmetry in the yaw torque; it is seen that two null points occur at $\psi = 60^{o}$ and $\psi = -120^{o}$. The second graph, which analyzes yaw torques into causal components, shows the reason for this; it is largely due to the 90-degree phase difference between the SOEP antenna and the EP5 torus torques.

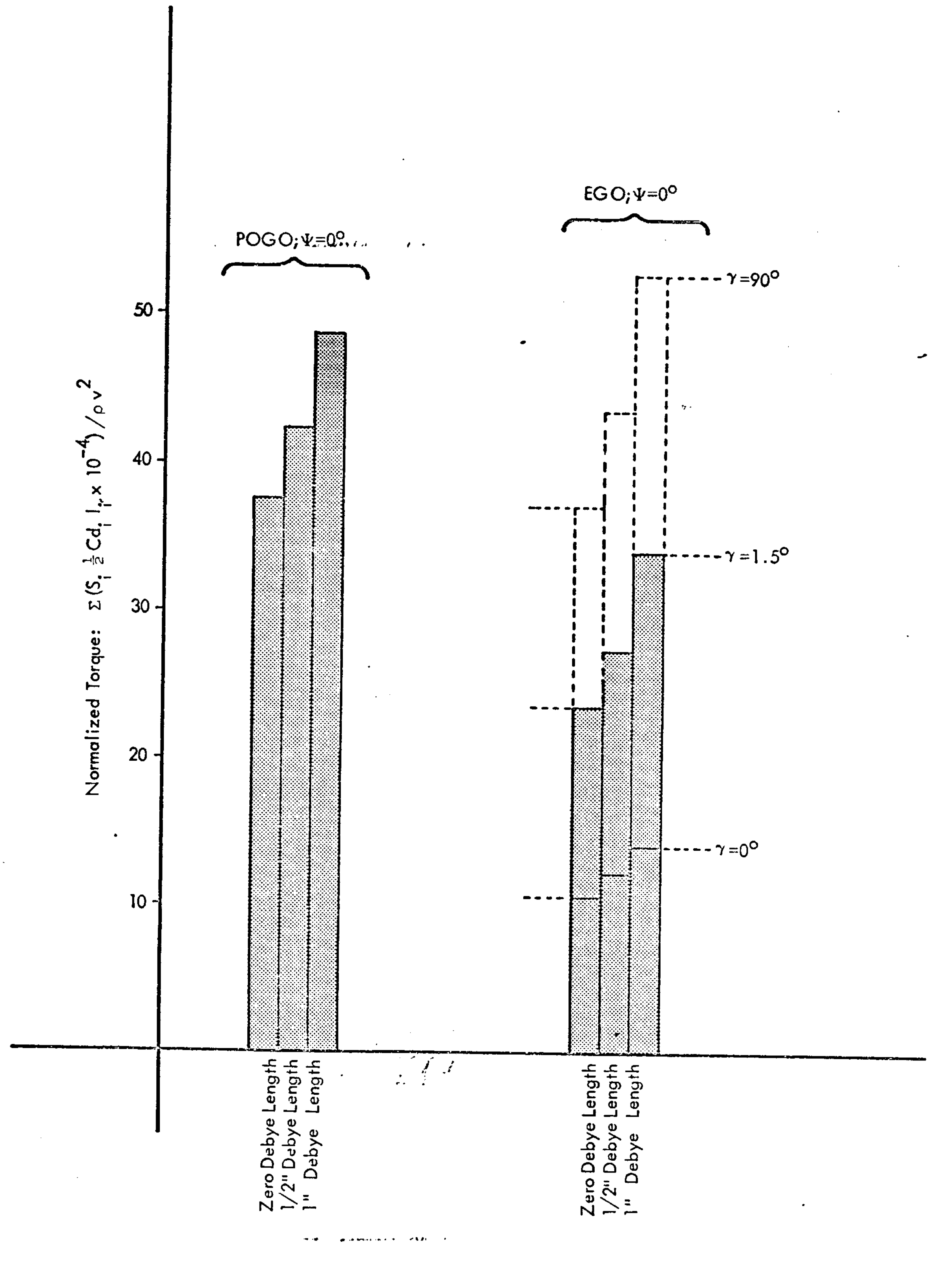


Figure 6 EFFECT OF DEBYE LENGTH ON AERODYNAMIC TORQUES

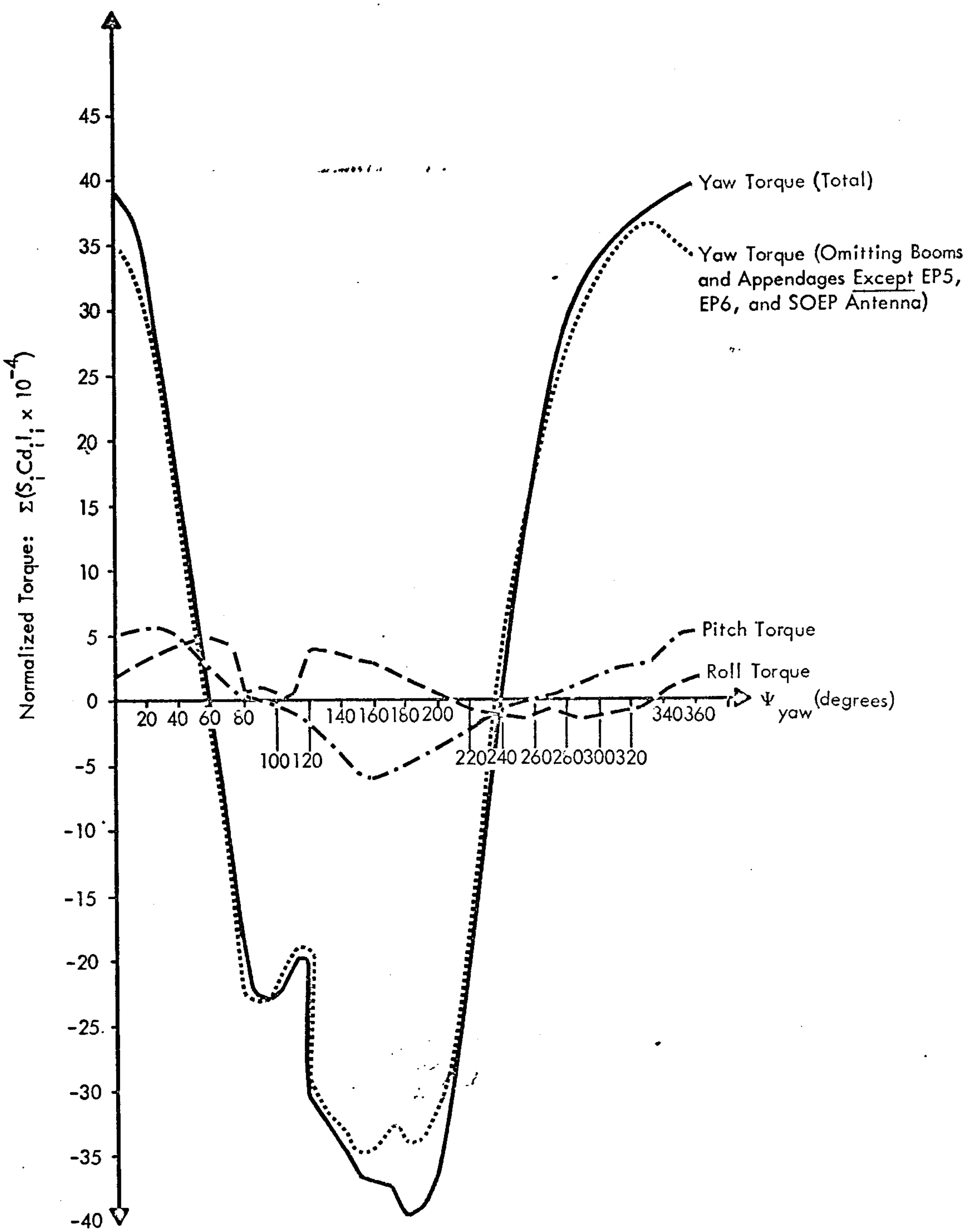


Figure 7 AERODYNAMIC TORQUE AS A FUNCTION OF YAW ANGLE

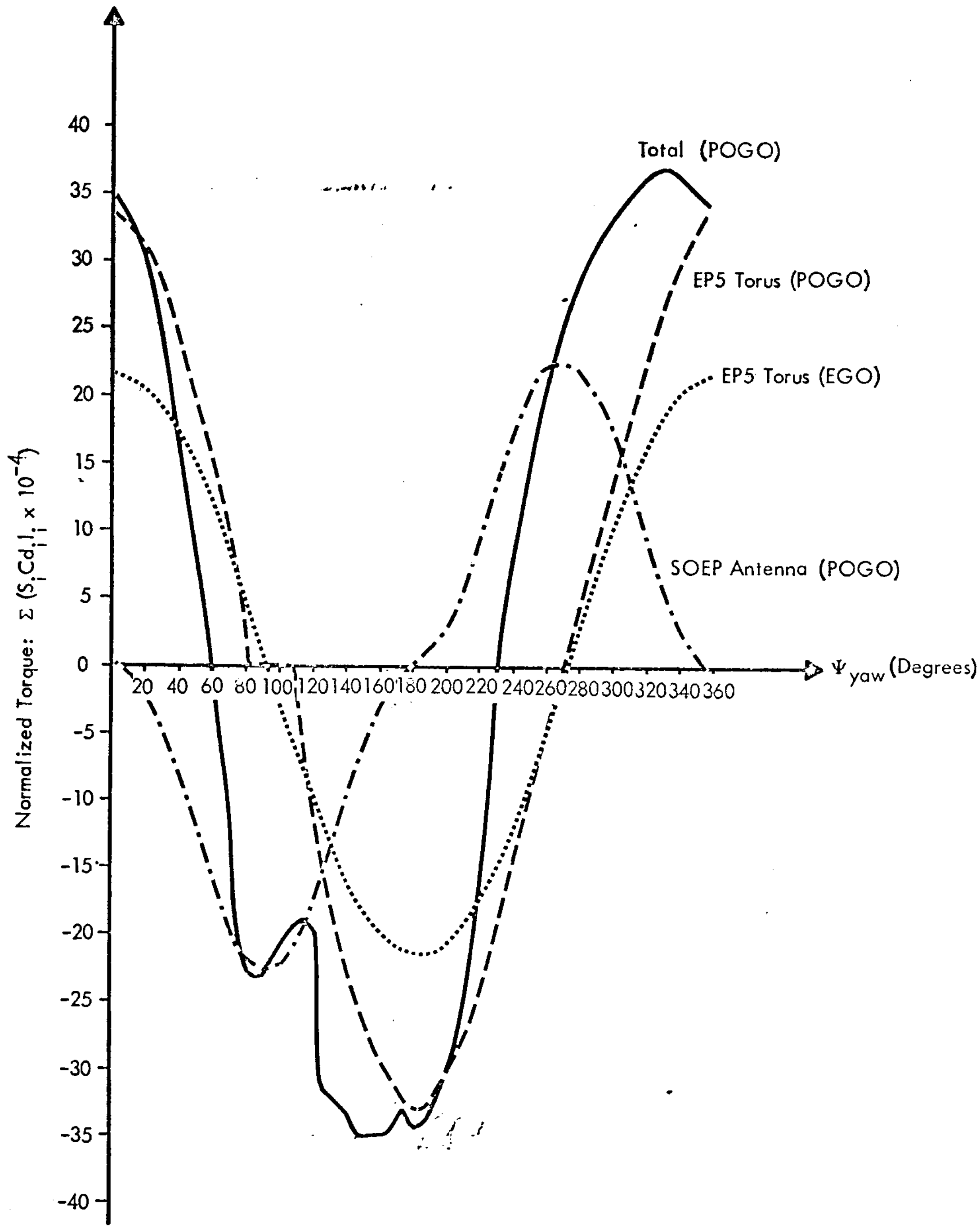


Figure 8 YAW TORQUE AS A FUNCTION OF YAW ANGLE

2. CYCLIC AND SECULAR DISTURBANCE TORQUES

The torques to which the spacecraft is subjected are of the following three kinds:

Cyclic	Control Torques
Cyclic Secular	Disturbance Torques

Since the inertia wheels have been designed conservatively with enough storage capacity to observe cyclic (i.e., self-canceling) torques from all courses over a complete orbit, gas jet firings will only be required to unload disturbance torques accumulating within each orbit and between successive orbits.

As a means of checking the validity of computed angular momenta per orbit during program debugging, an attempt was made to analyze the accumulation of angular momentum for roll, pitch, and yaw angles, separating cyclic from the secular components in each case. This done, it was then possible to make crude "slide-rule" estimates of noncanceling torques with which to confront corresponding momenta generated by the program. A "fringe benefit" of this analysis is in indicating the dependence of gas expenditure upon the orbital parameters and orientation.

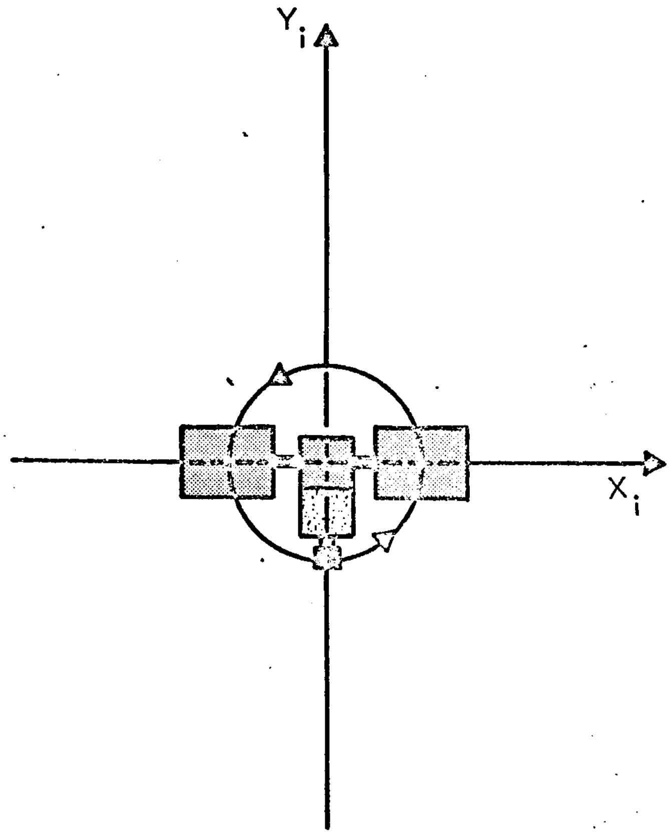
The problem in conducting this analysis was to find an intuitively easy means of translating torques generated in the satellite body coordinate system to an inertial system. The "closest" inertial (or rather, irrotational) system to the body coordinates appears to be that defined by the plane of the paddles, together with an axis perpendicular to it. When the satellite is properly controlled, this latter axis will always point to the sun.

When the satellite in this irrotational coordinate system is viewed looking from the sun, the following movements are observed during each orbital revolution:

- One 360-degree revolution of the paddles around the centroid of the satellite in the $x_i y_i$ plane of the inertial system
- A nutation of the spacecraft y-axis carrying it in a circle which touches the z_i-axis at one point and subtends a maximum angle of S' degrees to it half a revolution later. Hence, when $S' = 0^\circ, 180^\circ$, there is no nutation.

These two motions occur simultaneously, i. e., are superimposed on each other. One effect of the combined motions is to keep both y and z faces always hidden from the sun.

Both motions are illustrated below. The rate at which the two motions occur will be uniform for circular orbits. For noncircular orbits there will be a nonuniformity implying unequal dwell times at the various orientations; this asymmetry will be further increased as the argument of the projection of the sun vector in the orbital plane moves away from perigee or apogee.



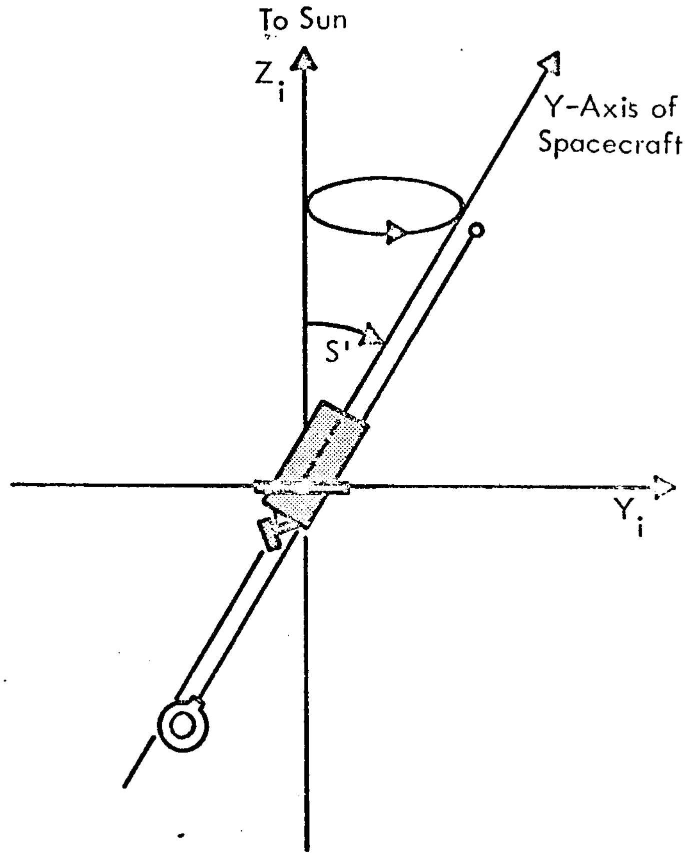
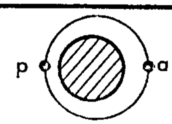
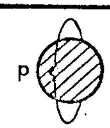
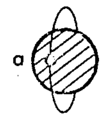
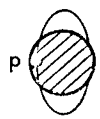
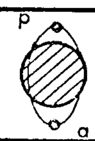
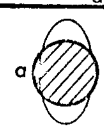


Table 2 shows the extent to which torques in the three dimensions (pitch, roll, and yaw) from three origins (aerodynamic, solar, and gravity-gradient) are self-canceling. The table presents these as a

Table 2
Degree of Torque Cancellation as a Function of Orbital Orientation (POGO)

Orbital Orientation		Aerodynamic			Solar			Gravity-Gradient		
		Roll	Pitch	Yaw	Roll	Pitch	Yaw	Roll	Pitch	Yaw
S' = 0°		Mostly Adds*	Adds	Mostly Adds*	Cancels	Adds	Cancels	Cancels	Adds	Adds
S' = 90° δ = 0°		Adds	Adds*	Cancels	Cancels	Adds	Adds	Adds	Adds	Cancels
S' = 90° δ = 90°		Adds	Adds*	Mostly Adds*	Mostly Cancels	Mostly Adds	Mostly Adds	Adds	Adds	Mostly Cancels
S' = 90° δ = 180°		Adds	Adds*	Cancels	Cancels	Adds	Adds	Adds	Adds	Cancels
S' = 45° δ = 0°		Adds	Adds	Partly Cancels	Partly Cancels	Adds	Partly Cancels	Partly Cancels	Adds	Partly Cancels
S' = 45° δ = 90°		Adds	Adds	Mostly Adds	Partly Cancels	Adds	Partly Cancels	Partly Cancels	Adds	Partly Cancels
S' = 45° δ = 180°		Adds	Adds	Partly Cancels	Partly Cancels	Adds	Partly Cancels	Partly Cancels	Adds	Partly Cancels

* Partly cancels if eccentricity = 0.

function of orbital position relative to the sun. This position, for a given orbital ellipse, has only two degrees of freedom:

- Inclination of the orbital to the sun vector; this is given by angle S'. Angle S' is 0^{o}, 180^{o} when orbital is normal to the sun, 90^{o} when edge-on.
- Argument of the projection of the sun vector in the orbital plane from perigee; this is given by angle δ.

The orbital orientations are indicated geometrically on the left of the table. The sun is looking normally into the paper in all cases.

A glance at the table shows that different symmetries hold for aerodynamic, solar, and gravity-gradient forces. This is not surprising, in view of the difference in orientation of the three forces.

For orbitals below 180 n. m. perigee, torques of aerodynamic origin, particularly yaw torques, are the major cause of gas expenditure. For normal POGO orbits of eccentricity in the neighborhood of 0.04 to 0.05, most aerodynamic drag occurs over a small sector of the orbit in the immediate neighborhood of perigee. This asymmetry eliminates most of the possibility for torque-cancellation. A marked exception is for orbitals edge-on to the sun with $\delta = 0^{o}$ or 180^{o}, which leads to self-cancellation of yaw torque. Partial cancellation of this component of torque occurs for S' at intermediate angles between 0^{o}

and 90°. For <u>circular</u> orbits, however, the corresponding uniformity of air drag leads to a cyclic cancellation of most aerodynamic torques.

Complicating the whole aerodynamic torque picture is the relation of torque to yaw angle; the function is not symmetrical with respect to positive and negative angles.

Comparatively speaking, solar torques are not important for POGO orbits.

Gravity-gradient torques show a higher degree of symmetry than aerodynamic, owing to the fact that the force does not vary much over the orbit. Thus, for $S' = 90^{\circ}$, yaw gravity-gradient torque (which is the largest of the three) mostly cancels, no matter what the value of δ may be.

The preceding analysis is complicated by the occurrence of eclipses, which introduce an asymmetric influence. This will be greatest for edge-on orbitals with $\delta = 90^{\circ}$, 270°. The effect of eclipses on torque-cancellation follows mostly from the loss of control of the yaw angle. During the period of eclipse, there is no easy way of determining what happens to the yaw angle; hence, it is difficult to conclude what effect eclipses would have on the analysis presented in Table 2.

In conclusion, it appears that for POGO orbitals, the "best case" from the point of view of gas consumption will occur for edge-on orbits with $\delta = 0^{\circ}, 180^{\circ}$, since this leads to cancellation of some aerodynamic yaw and all gravity-gradient yaw torques.

3. COMPUTER PARAMETRIC STUDY

Gas budget expenditures were obtained for single orbitals for four orbital inclination values (0°, 45°, 90°, and 135°) and three argument of perigee values (0°, 90°, and 180°). These are presented in factorial form below. Shown in parentheses are corresponding values obtained by suppressing eclipses. All orbitals had a perigee of 180 n. m.

Argument of Perigee	Sun Inclination (to orbital plane)			
	0°	45°	90°	135°
0°	0. 209	0. 581 (0. 543)	0. 349 (0. 206)	0. 625 (0. 594)
90°	0. 209	0. 365 (0. 258)	0. 147 (0. 129)	0. 160 (0. 096)
180°	0. 209	0. 517 (0. 520)	0. 105 (0. 104)	0. 490 (0. 621)

It is emphasized that the pattern of expenditures obtained is dependent upon perigee altitude which drastically affects the torque contribution of aerodynamic origin.

Certain trends can be detected in the above data:

- Sun inclinations of 0^{o} and 90^{o} are associated with lower gas consumption, probably because very small gravity-gradient torques are developed under these circumstances.

- Argument of perigee value of 90^{o} seems to be associated with smaller gas expenditures. This may be because the aerodynamic null point (see earlier part of this section above) is brought into coincidence with that region of the orbital in the neighborhood of perigee where most of the aerodynamic drag occurs.

- Suppressing eclipses reduces gas expenditure. This is probably because the yaw angle is in control during all 360^{o} of each orbital; the consequent symmetry leads to cancellation of those torque components which are cyclic. The magnitudes of the differences between eclipse and no eclipse orbitals give some slight indication of the effects of the convention adopted by this program of holding the yaw angle constant during an eclipse.

APPENDIX A

CALCULATION OF ORBITAL PARAMETERS OF INITIAL ORBIT

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CALCULATION OF ORBITAL PARAMETERS OF INITIAL ORBIT

1. INTRODUCTION

Information concerning the properties of the satellite orbital must be computed from the injection parameters. The information given and the information required are as follows:

Time of Injection	Orbital Inclination (ξ)
Longitude of Injection Point	Argument of Ascending Node (β)
Latitude of Injection Point	Semimajor Axis (a)
Altitude of Injection Point	Argument of Perigee (λ)
Velocity at Injection	Eccentricity (e)
Azimuth of Orbit at Injection	Sun Angle from Autumnal Equinox (S)
Flight Path Angle at Injection	
Date of Injection	

The required orbital parameters are with reference to the ecliptic plane. The computational flow to obtain these orbital parameters is shown in Figure A-1.

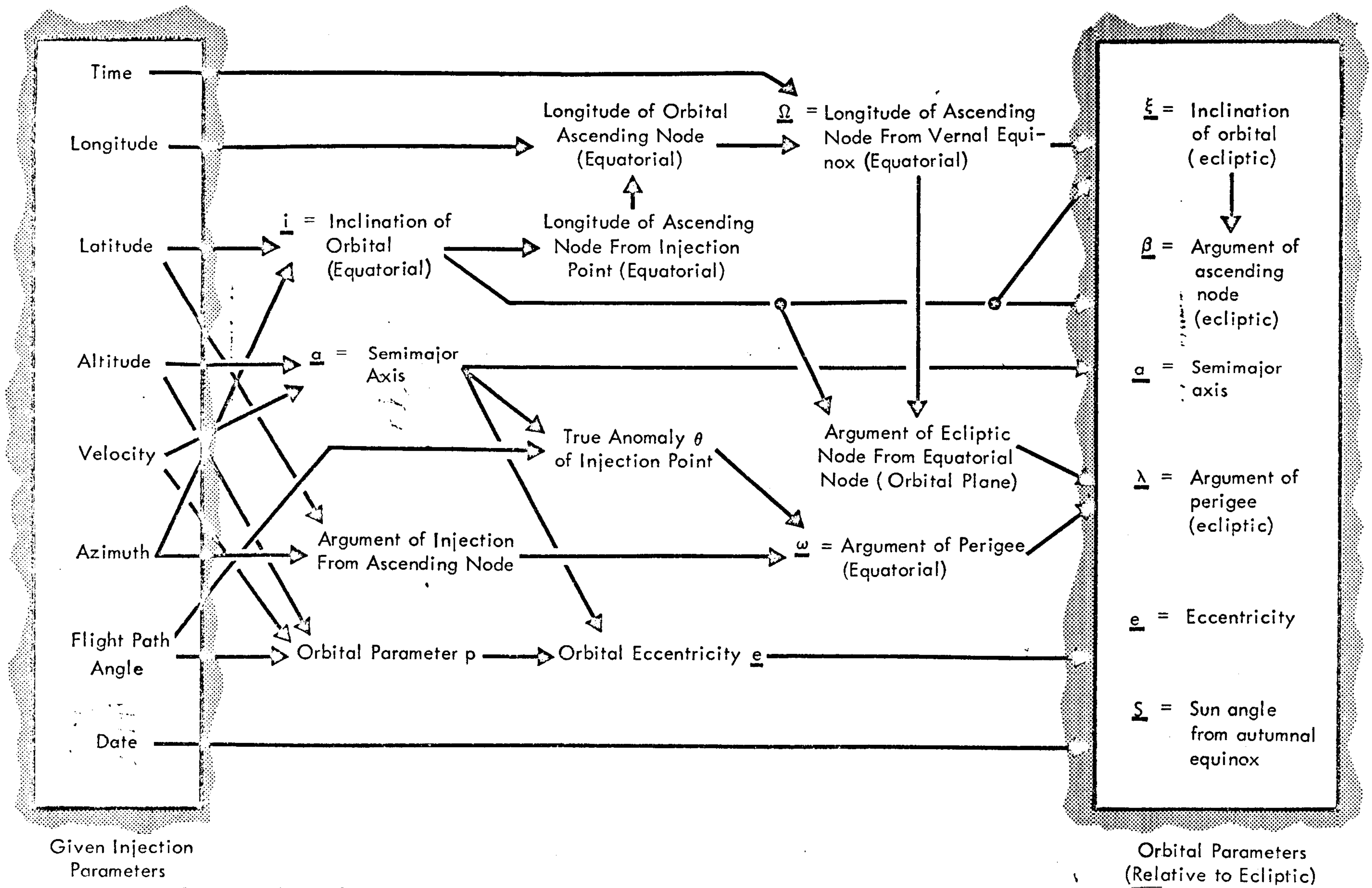


Figure A.1 COMPUTATIONAL FLOW OF REQUIRED ORBITAL PARAMETERS

It will be seen that the degrees of freedom supplied by the injection parameters are sufficient. They are distributed as follows:

3	$\left\{\begin{matrix}\xi\\ \beta\\ \lambda\end{matrix}\right\}$	Locate Orbital Plane in Space	Needed for OGO Program
2	$\left\{\begin{matrix}a\\ e\end{matrix}\right\}$	Size and Shape of Orbital	
1	S	Sun Location	
1	H	Earth Hour Angle	Not Needed for OGO Program
1	θ	Argument of Injection Point	
8	=	Total Degrees of Freedom	

Also needed for computing orbital perturbations are the orbital parameters with respect to the equatorial plane. These are obtained as intermediate steps in the above computations. These parameters are:

Ω = argument of ascending node from vernal equinox

ω = argument of perigee from vernal equinox

i = inclination of orbital plane.

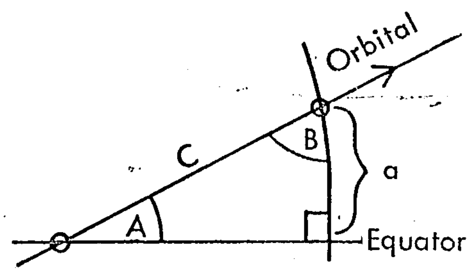
2. COMPUTING INCLINATION OF ORBIT

$\cos A = \sin B \cos a$

or

$\cos(\text{inclination}) = \sin(\text{azimuth}) \cos(\text{latitude})$

$\text{inclination} = \text{arc} \cos\{\sin(\text{azimuth}) \cos(\text{latitude})\}$.



At the equator, $\cos(\text{latitude}) = \cos(0) = 1$, so inclination $= \text{arc} \cos\{\sin(\text{azimuth})\}$

$= \text{arc} \cos\{\cos(90^\circ - \text{azimuth})\}$

$= 90^\circ - \text{azimuth}$.

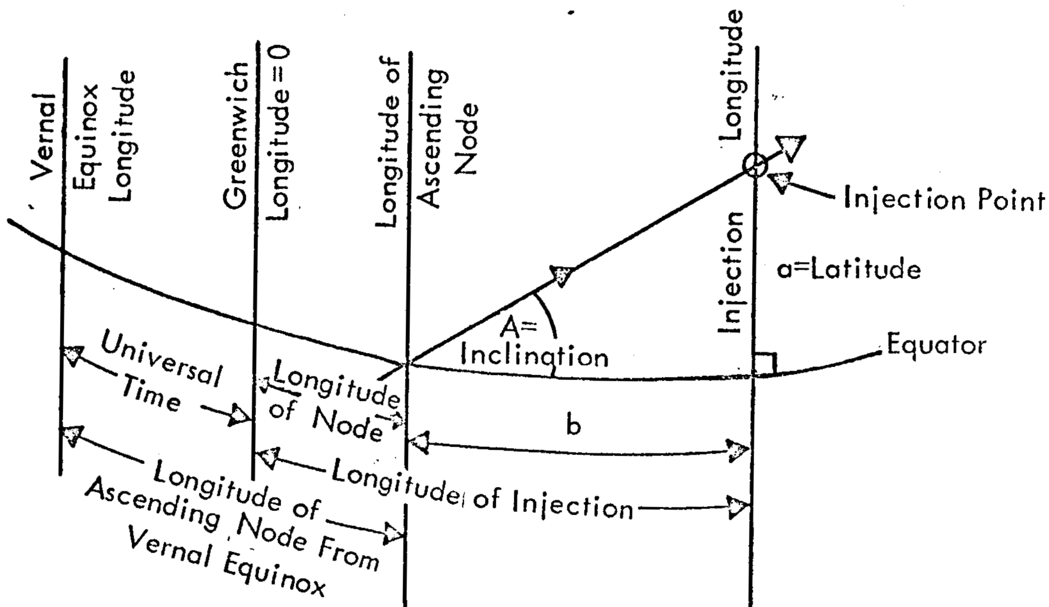
At all other latitudes, $\cos(\text{latitude}) < 1$, so

inclination $> (90^\circ - \text{azimuth})$ i.e., azimuth + inclination $\geq 90^\circ$.

If latitude $= 90^\circ$, $\cos(\text{latitude}) = 0$, cos inclination $= 0$, and inclination $= 90^\circ$.

Hence, all three angles of $\Delta = 90^\circ$, azimuth + inclination $= 180^\circ$. In conclusion, $(90^\circ - \text{azimuth}) \leq \text{inclination} \leq 90^\circ$.

3. COMPUTING LONGITUDE OF ASCENDING NODE FROM VERNAL EQUINOX (IN EQUATORIAL PLANE)



Clearly, once b is obtained, the longitude of ascending node is obtained by successive additions of angles.

We have

$$\sin b = \tan(\text{latitude}) \cdot \text{ctn}(\text{inclination}) = \frac{\tan(\text{latitude})}{\tan(\text{inclination})}.$$

We may note that

- If latitude = inclination, sin (b) = 1, b = 90°.
- Considering the rearranged form:

  tan (latitude) = tan (inclination) · sin b.

  For a given inclination, latitude will be a maximum when b = $90^{\circ} \rightarrow$ sin b = 1.0, at which point latitude = inclination.

There are sign difficulties in computing b owing partly to the split-circle of longitude measure and partly due to the fact that bearings made south of the equator are still with reference to the North Pole.

4. COMPUTATION OF ARGUMENT OF PERIGEE

This computation proceeds by the following steps:

- Find semimajor axis of orbit from $1/a = 2/r - V^2/\mu$
- Find $q = r/2a$
- Find true anomaly of injection point, θ, from $\tan(\theta - \gamma) = \tan\gamma / (1-2q)$
- Argument of injection (aoi) is obtained from cos (azimuth) = tan (latitude) · tan (aoi)
- Finally, argument of perigee ω, is given by $\omega = (\text{aoi}) - \theta$.

Note that flight path angle is

$$\text{positive} = 0 \leq \theta \leq 180$$

$$\text{negative} = 180 \leq \theta \leq 360 .$$

$$\left[\text{From the relationship } \gamma = \text{arc tan}\left\{\frac{e}{\sqrt{1 - e^2}} \sin E\right\}\right].$$

5. COMPUTATION OF ORBITAL CHARACTERISTICS

This section deals with the computation of:

- Orbital parameter, p
- Orbital eccentricity, e
- Perigee radius, r_0
- Apogee radius, r_{180}
- Maximum flight path angle, γ_{max}
- θ corresponding to maximum flight path angle.

The orbital parameter, p, may be obtained from the following relationship:

$$rv \cos \gamma = \sqrt{\mu p}$$

$$p = \frac{(rv \cos \gamma)^2}{\mu}.$$

Eccentricity, e, is then obtained from

$$e = \sqrt{1 - p/a}$$

where

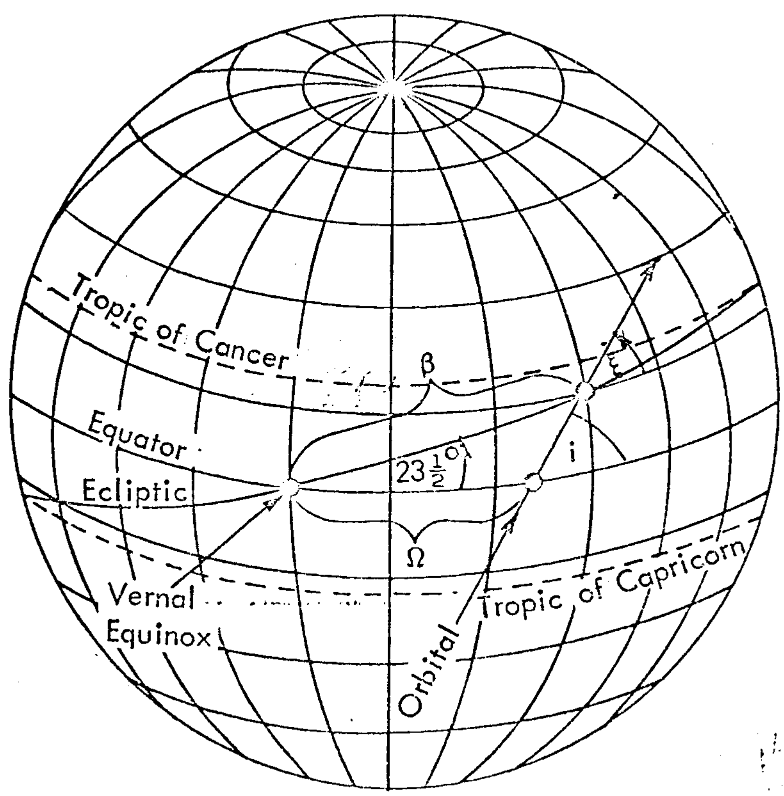
a = semimajor axis.

Perigee radius or apogee radius are obtained from

$$r_0 = a(1 - e)$$

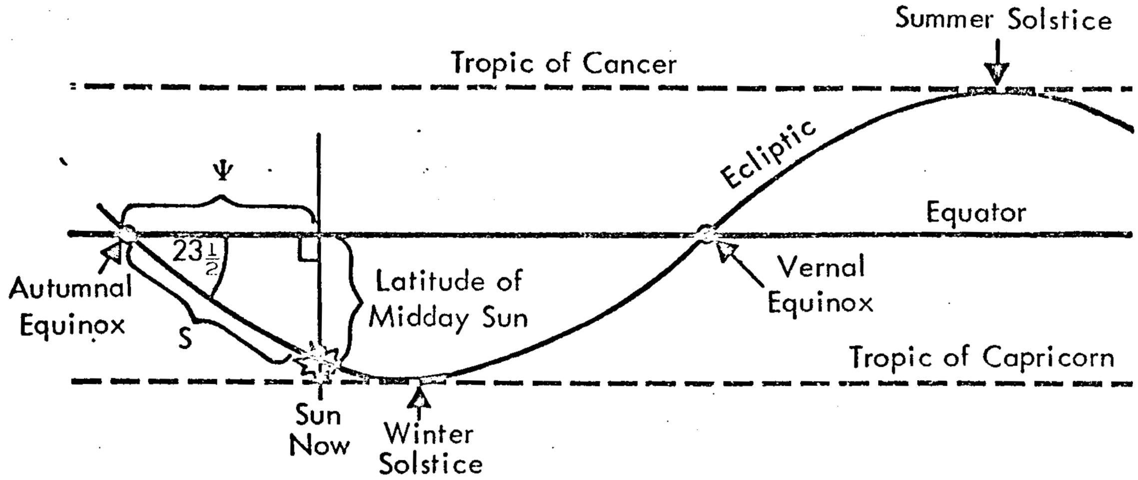
$$r_{180} = a(1 + e).$$

6. DETERMINATION OF ORBITAL PARAMETERS RELATIVE TO ECLIPTIC PLANE



(1) Location of Argument of Midday Sun From Autumnal Equinox Along Ecliptic

The argument must be taken from the autumnal and not the vernal equinox since, though we are in the ecliptic plane, our inertial system is geocentric and not heliocentric. This creates a 180° phase difference.



This argument corresponds to STL's, S, and is given by the proportion of the year which has elapsed since the earth passed through the vernal equinox (or time since sun passed through autumnal equinox). It is therefore given by

$$S = 360 \left(\frac{\text{days since vernal equinox}}{365 - 1/4} \right)$$

(2) Latitude of Midday Sun

The latitude of midday sun is given by:

$$\sin(\text{latitude}) = \sin(\text{inclination}) \cdot \sin(-S)$$
$$= 0.3987 \cdot \sin(-S).$$

(3) Angle of Inclination of Orbital to Ecliptic

$$\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a$$

where

$$A = \xi$$
$$B = (180-i)$$
$$C = 23\text{-}1/2$$
$$a = \Omega$$

$$\cos \xi = -\cos(180-i) \cdot \cos 23\text{-}1/2 + \sin(180-i) \cdot \sin 23\text{-}1/2 \cdot \cos \Omega$$
$$= 0.9171 \cos(i) + 0.3987 \sin(i) \cdot \cos \Omega$$

since

$$\cos(180-i) = -\cos i$$
$$\sin(180-i) = \sin i.$$

When the orbit is inclined in the reverse direction, the angle obtained is the 180° complement:

$$\cos(180-\xi) = -\cos i \cos 23\text{-}1/2 + \sin i \sin 23\text{-}1/2 \cos \Omega$$

$$\cos \xi = 0.9171 \cos i - 0.3987 \sin i \cos \Omega .$$

(4) Argument of the Ecliptic Ascending Node From the Vernal Equinox ($= \beta$)

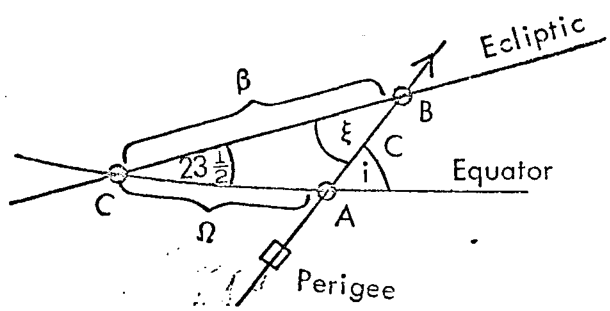
$$\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a$$

$$\cos(180-i) = -\cos \xi \cdot \cos 23\text{-}1/2 + \sin \xi \cdot \sin 23\text{-}1/2 \cdot \cos \beta$$

$$\cos i = 0.9171 \cos \xi - 0.3987 \sin \xi \cdot \cos \beta$$

$$0.3987 \sin \xi \cdot \cos \beta = 0.9171 \cos \xi - \cos i$$

$$\cos \beta = \frac{0.9171 \cos \xi - \cos i}{0.3987 \sin \xi} .$$



Had the ecliptic crossed the equator in the reverse direction from C, the above equation becomes modified to

$$\cos \beta = \frac{\cos i - 0.9171 \cos \xi}{0.3987 \sin \xi}$$

(5) Argument of Perigee Along Ecliptic From the Vernal Equinox ($= \lambda$)

Referring to the above diagram, it is seen that:

$$\lambda = w - c$$

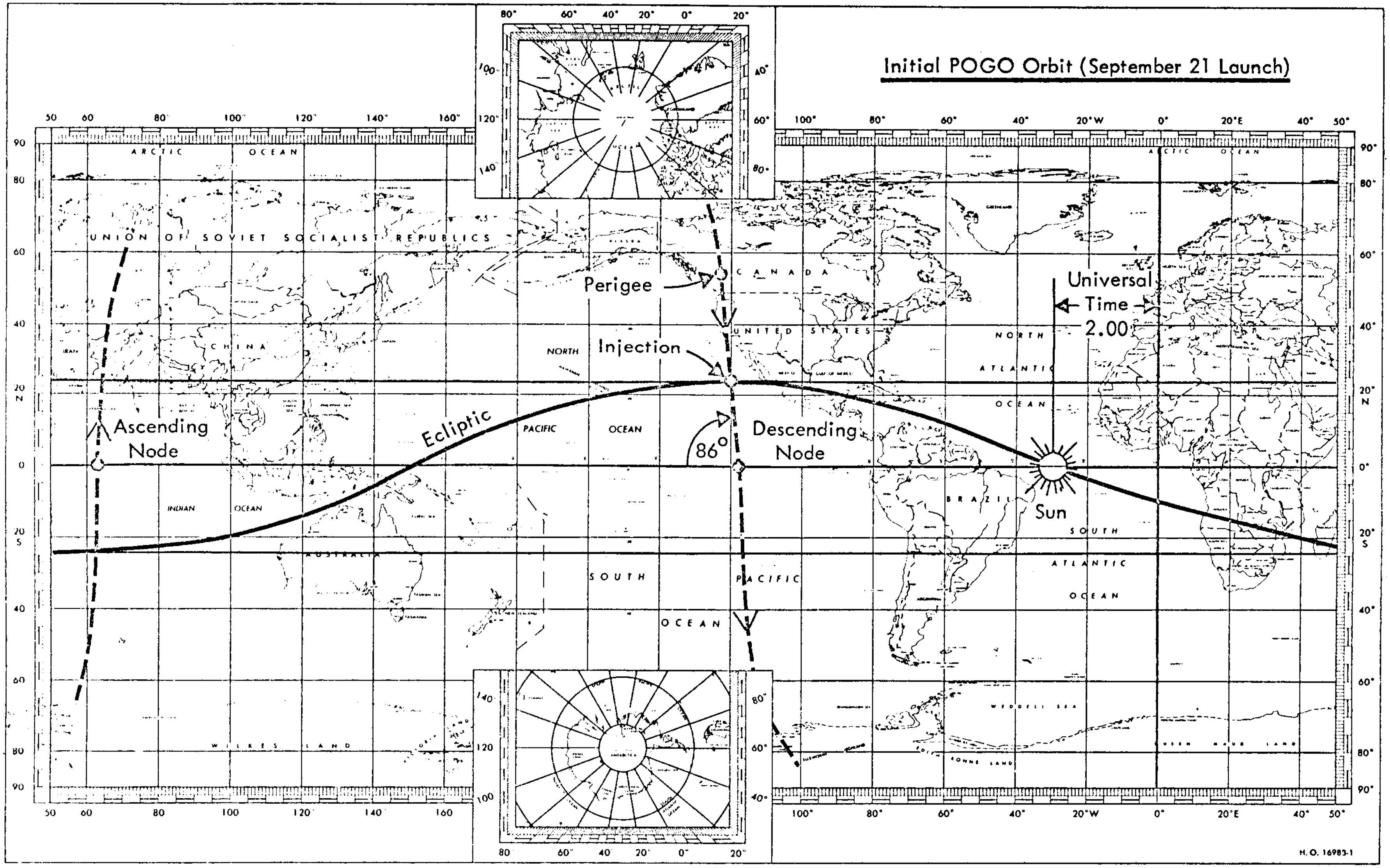
($= w + c$ if ecliptic crosses in reverse direction)

$$\cos C = -\cos A \cdot \cos B + \sin A \cdot \sin B \cdot \cos c$$

$$0.9171 = -\cos(180-i) \cdot \cos \xi + \sin(180-i) \cdot \sin \xi \cos c$$

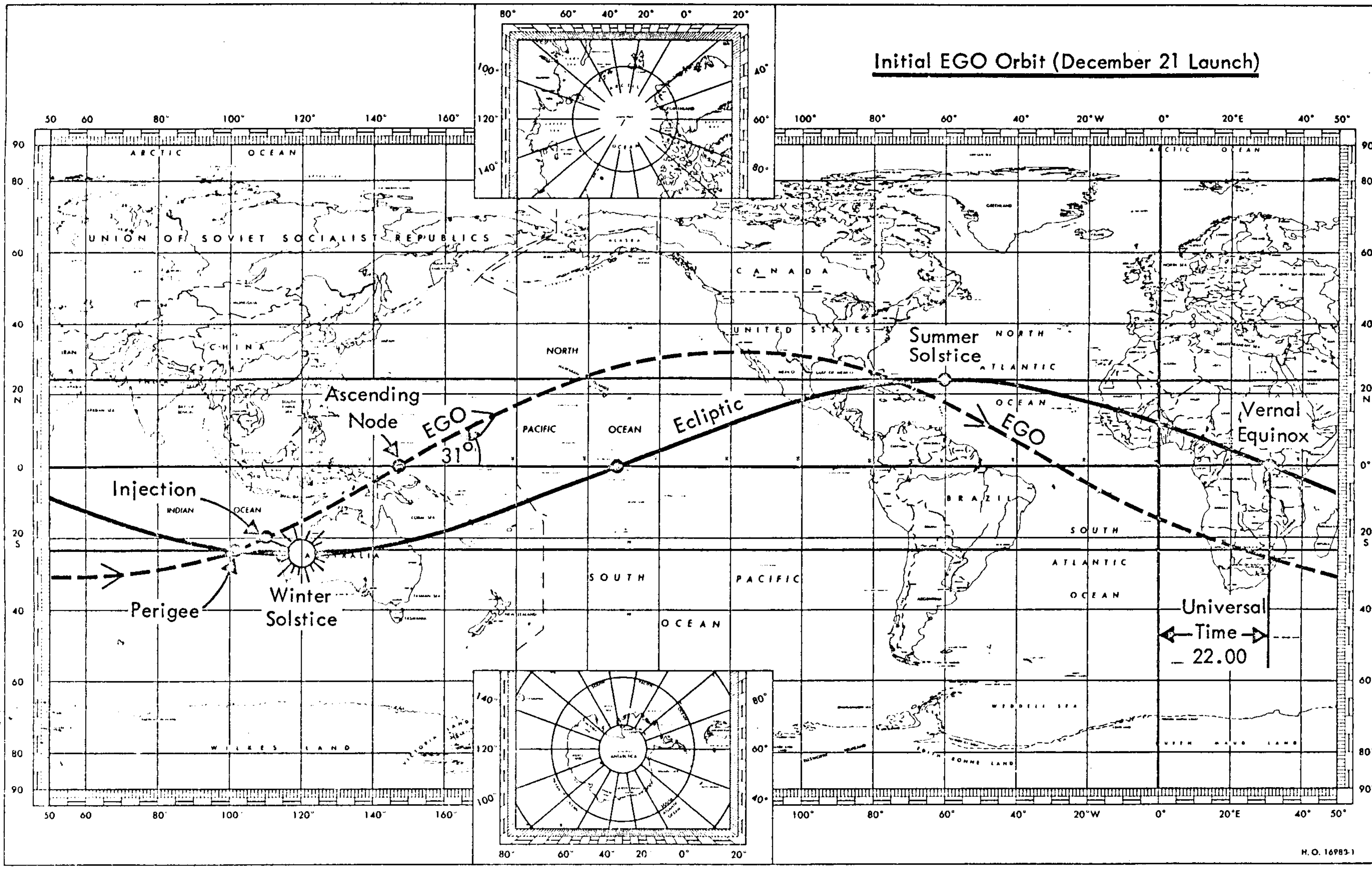
$$= \cos i \cdot \cos \xi + \sin i \cdot \sin \xi \cdot \cos c$$

$$\cos c = \frac{0.9171 - \cos i \cos \xi}{\sin i \cdot \sin \xi}.$$



Initial POGO Orbit (September 21 Launch)
Perigee
Injection
Universal
Time
2.00
Ascending
Node
Descending
Node
Ecliptic
86°
Sun
UNION OF SOVIET SOCIALIST REPUBLICS
CHINA
CANADA
UNITED STATES
BRAZIL
AUSTRALIA
ARCTIC OCEAN
NORTH ATLANTIC OCEAN
SOUTH ATLANTIC OCEAN
NORTH PACIFIC OCEAN
SOUTH PACIFIC OCEAN
INDIAN OCEAN
WILKES LAND
H. O. 16983-1

Initial EGO Orbit (December 21 Launch)



BAARINC

APPENDIX B

COMPUTER PROGRAM PRINTOUT

```
C      PROGRAM ORBIT
       COMMON PSI,PSIV,PSIY,P1,PHI,PHIP,IUV,IUP,XG,YG,ZG
      1,H5,H6,H7,COMP,AEL,BEL,H,EEL,W,FL,CEL,PADW,XX,TPSI,TPHI,PAD,ZI,SX
      2,SY,FX,FY,FZ,TA(3)
      3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
      4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
      510,4),OPEP(6),TORQ(40,3),BL
      6,AY,AZ,AB5,AB6,AF5,AOP,ACS,V,ATMO(15), VEL,TS(3),OP(3),ALT
      7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA ,NEGO ,NDAYS
      8,TG,CONS,GTHX,GTHY,GTHZ,XXI,YYI,ZZI,XSUM,YSUM,ZSUM
       DIMENSION T(365)
       DIMENSION AMAT(3,3),AIMAT(3,3)
       DIMENSION BMAT(3,3), CMAT(3,3)
       DIMENSION TG(3),TASUM(3),TSSUM(3),TGSUM(3)
       DIMENSION THRSTX(20),THRSTY(20 )
       DIMENSION TAINT(3,365),TSINT(3,365),TGINT(3,365)
7676   FORMAT(1HO16HNUMBER OF DAYS = 16)
240    FORMAT(1HO5HAERO 3E16.8)
     1 FORMAT(20X,4F10.0)
     2 FORMAT(16X,4E16.8)
 3     FORMAT(20X,4I5)
1001   FORMAT(10X,6F10.3)
1002   FORMAT(10X3E20.8)
       PI=3.1415926
       RAD=0.017453293
       DEG=57.295779
7777   CONTINUE
       READ(5,3)NOSHAD,NEGO,NDAYS
       READ(5,1001)FNORB
       READ(5,3)IAIR,ISUN,IGRAV
       READ(5,3) ITORTA
       NOSHAD = NOSHAD+1
       READ (5,1001)F4Y,F4X,CANT,S2,C3,C1
       READ (5,1001)BX,COPEP,OPEP
       READ (5,1001)B6,S6,B5,C5,F5X,F5Y,H,EEL,AEL,BEL,CEL,FL,W,SGMA,SGMAP
       READ(5,1001) XG,YG,ZG
       READ (5,1)AY,AZ,AOP,AP,AB6,AX,AB5,AF5,AC5
       READ (5,1001)OP
       READ (5,1002)ATMO
       READ (5,1002)V
       READ(5,1001)THRSTX
       READ(5,1001)THRSTY
C      A IS SEMIMAJOR AXIS,E IS ECCENTRICITY,XI IS INCLINATION OF ORBITAL PLANE
C      FROM ECLIPTIC,S IS ANGLE OF SUN VECTOR FROM NEG VERNAL EQUINOX,OMEGA
C      IS ANGLE FROM LINE OF NODES TO PERIGEE
       READ(5,1)XXI,YYI,ZZI
       READ(5,1)GTHX,GTHY,GTHZ
       READ(5,3)NORBIT,NINTER,IPRINT
       READ(5,1) ERRKEP
       READ (5,2) GM,RE
       FINTER=NINTER
       SGASX=0.
       SGASY=0.
       DO 1000 IORB=1,NORBIT
       NODAYS = NDAYS*IORB
       READ(5,525)A
  525  FORMAT(20X,E16.8)
       READ(5,1) E,XI,S
       READ(5,1)OMEGA,BETA
       READ(5,1)ALPHA1,ALPHA2,ALPHA3,ALPHA4
       WRITE(6,53)
```

```
   53 FORMAT(1H1,15X,40HINPUT INFORMATION AND ORBITAL PARAMETERS)
      WRITE(6,59) A,E
   59 FORMAT(1H0//1H0,30X,18HELLIPSE PARAMETERS/1H0,30X,2HA=E16.8,10X,2H
     1E=E16.8)
      WRITE(6,60)GM,RE
   60 FORMAT(1H0//1H0,30X,9HCONSTANTS/1H0,25X,3HMU=E16.8,10X,3HRE=E16.8)
      WRITE(6,61)NORBIT,NINTER,ERRKEP
61    FORMAT(1H016HNUMBER OF ORBITSI6/1H033HNUMBER OF INTERVALS IN EACH
     1ORBITI15/1H017HKEPLER ERROR TESTF10.8)
      WRITE(6,55) BETA,OMEGA,S,XI
   55 FORMAT(1H0//1H039HORBIT ANGLES BETA,OMEGA,S,XI IN DEGREES4(F10.4))
      WRITE(6,203)ALPHA1,ALPHA2,ALPHA3,ALPHA4
203   FORMAT(1H015X36HECLIPSE ANGLES MEASURED FROM PERIGEE4F10.4)
      ALPHA1=ALPHA1*RAD
      ALPHA2=ALPHA2*RAD
      ALPHA3=ALPHA3*RAD
      ALPHA4=ALPHA4*RAD
      BETA=RAD*BETA
      XI=RAD*XI
      S=RAD*S
      OMEGA=RAD*OMEGA
      IF(ALPHA2.LT.ALPHA1)ALPHA2=ALPHA2+2.*PI
      IF(ALPHA3.LT.ALPHA2)ALPHA3=ALPHA3+2.*PI
      IF(ALPHA4.LT.ALPHA3)ALPHA4=ALPHA4+2.*PI
      DO 247 I=1,3
      TASUM(I)=0.
      TSSUM(I)=0.
      TGSUM(I)=0.
      TG(I)=0.
      TA(I)=0.
      TS(I)=0.
247   CONTINUE
      COSSPR=SIN(XI)*SIN(S-BETA  )
      IF(COSSPR.GT.0.) GO TO 72
      IF(COSSPR.LT.0.) GO TO 75
      SPR=PI/2.
      GO TO 73
72    SPR=ATAN(ABS((1.-COSSPR**2)**.5/COSSPR))
      GO TO 73
75    SPR=PI-ATAN(ABS((1.-COSSPR**2)**.5/COSSPR))
 73   CONTINUE
      PSPR=SPR*DEG
      P=(2.*PI*A**1.5)/GM**.5
      PMIN=P/60.
    6 TINT=P/FINTER
      DELTIM=0.
      WRITE(6,66)
   66 FORMAT(1H1,30X,63HCOMPUTED ORBIT PARAMETERS-THAT REMAIN CONSTANT T
     1HROUGHOUT ORBIT)
      WRITE(6,67)PSPR,SPR
 67   FORMAT(1H0//1H024HSUN VECTOR ANGLE SPRIME=E16.8,10H DEGREES (E16.8
     1,9H RADIANS))
      WRITE(6,50)P,PMIN
   50 FORMAT(1H0///1H018HPERIOD IN SECONDS=E16.8,8X,18HPERIOD IN MINUTES
     1=E16.8)
      DO 100 N=1,NINTER
      FACTOR=1.
      INDECL=1
      TI=N
      GO TO (425,220),ITORTA
  425 T(N)=TI*TINT
      TMIN=T(N)/60.
```

```
      AMEAN=(2.*PI*T(N))/P
C     ITERATIVE PROCEDURE FOR KEPLERS EQUATION
      CALL KEPLER(AMEAN,E,ERRKEP,E2,KC)
C     COMPUTE TRUE ANOMALY ANGLE XNU
   16 A1=SIN(E2/2.)
      A2=COS(E2/2.)
      IF(A2)9,10,9
   10 IF(A1)11,11,12
   11 ANGLE=3.*PI/2.
      GO TO 15
   12 ANGLE=PI/2.
      GO TO 15
    9 A3=A1/A2
      CALL QUADCK(A1,A2,A3,J)
      B=((1.+E)/(1.-E))**.5
      TANGLE=B*A3
      ANGLE=ATAN(ABS(TANGLE))
      CALL CORANG(ANGLE,J,TANG)
      ANGLE=TANG
 15   XNU=2.*ANGLE
      GO TO 426
 220  CONTINUE
      XNU=360./FINIER*TI*RAD
      A1=SIN(XNU/2.)
      A2=COS(XNU/2.)
      IF(ABS(A2)-.00001) 427,428,427
  428 IF(A1) 429,429,430
  429 ANGLE=3.*PI/2.
      GO TO 431
  430 ANGLE=PI/2.
      GO TO 431
  427 A3=A1/A2
      CALL QUADCK(A1,A2,A3,J)
      B=((1.-E)/(1.+E))**.5
      TANGLE=B*A3
      ANGLE=ATAN(ABS(TANGLE))
      CALL CORANG(ANGLE,J,TANG)
      ANGLE=TANG
  431 E2=2.*ANGLE
      AMEAN=E2-E*SIN(E2)
      T(N)=P*AMEAN/(2.*PI)
  426 DAMEAN=AMEAN*DEG
      DE2=E2*DEG
      DXNU=XNU*DEG
      R=A*(1.-E*COS(E2))
      ALT=(R-RE)/6076.1
      VEL=(GM*(2./R-1./A))**.5
      VRAD=((  GM**.5  )*E*SIN(XNU))/(A*(1.-E**2))**.5
      VPERP=(GM**.5)*(1.+E*COS(XNU))/(A*(1.-E**2))**.5
   17 TANGAM=(E*SIN(E2))/(1.-E**2)**.5
      COSGAM= ((A**2*(1.-E**2))/(R*(2.*A-R)))**.5
      SINGAM=TANGAM*COSGAM
      CALL QUADCK(SINGAM,COSGAM,TANGAM,K)
      ANGLE = ATAN(ABS(TANGAM))
      CALL CORANG(ANGLE,K,TANG)
      GAMA=TANG
 213  CONTINUE
C     ORBITAL PARAMETERS COMPLETED,FIND VEHICLE PARAMETERS
      IF(ABS(COS(S-BETA)).LT..000001) GO TO 37
 18   TANETA=(COS(XI))*(SIN(S-BETA))/(COS(S-BETA))
      DENOM=SQRT((COS(S-BETA  ))**2+(COS(XI))**2*(SIN(S-BETA  ))**2)
```

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```
      COSETA=-COS(S-BETA )/DENOM
      SINETA=-COS(XI)*SIN(S-BETA )/DENOM
      ANGLE=ATAN(ABS(TANETA))
      CALL QUADCK(SINETA,COSETA,TANETA,KI)
      CALL CORANG(ANGLE,KI,TANG)
      ETA=TANG
 218  CONTINUE
      GO TO 38
 37   IF(SIN(S-BETA).LT.0.) ETA=3.*PI/2.
      IF(SIN(S-BETA).GT.0.) ETA=PI/2.
 38   CONTINUE
      PGAM=GAMA*DEG
      PETA=ETA*DEG
      TESTNU=XNU
      IF((ALPHA1.EQ.ALPHA2).AND.(ALPHA2.EQ.ALPHA3)) GO TO 85
      IF(ALPHA1.GT.TESTNU)TESTNU=TESTNU+2.*PI
      IF(ALPHA1.LE.TESTNU.AND.TESTNU.LE.ALPHA2)GO TO 81
      IF(ALPHA2.LE.TESTNU.AND.TESTNU.LE.ALPHA3)GO TO 82
      IF(ALPHA3.LE.TESTNU.AND.TESTNU.LE.ALPHA4) GO TO 83
      GO TO 85
C PENUMBRA 1
   81 MPEN=1
      CALL PNBRA(ALPHA1,TESTNU,ALPHA2,E,MPEN,FACTOR)
      GO TO 84
C PENUMBRA 2
   83 MPEN=2
      CALL PNBRA(ALPHA3,TESTNU,ALPHA4,E,MPEN,FACTOR)
      GO TO 84
   82 CONTINUE
C ECLIPSE
      INDECL=2
      FACTOR=0.
      GO TO 85
   84 CONTINUE
   85 CONTINUE
C     ALPHA=ANGLE FROM LINE OF NODES TO SATELLITE POSITION
      ALPHA=OMEGA+XNU
      PAQPHA=ALPHA*DEG
      GO TO (251,252),INDECL
C     COMPUTATION OF YAW ANGLE PSI
252   IF(N.GT.1)GO TO 288
  251 CONTINUE
      IF(ABS(COSSPR).LT..000001) GO TO 23
C     CHECK IF COS S PRIME IS 1 OR -1
      CHK=1.-ABS(COSSPR)
      IF(ABS(CHK)-.000001)33,33,22
 22   TANPSI=-SIN(SPR)/COSSPR*SIN(ALPHA-ETA)
C     TEST TO DETERMINE QUADRANT OF PSI,PSI IN QUAD 3 OR 4 WHEN ALPHA-
C     ETA IN 1 AND 2.PSI IN 1 OR 2 WHEN ALPHA-ETA IN 3 OR 4.
      TEST=SIN(ALPHA-ETA)
  20  IF(TEST)19,30,30
 30   IF(TANPSI.GT..0) KQ=3
      IF(TANPSI.LT..0) KQ=4
      GO TO 21
 19   IF(TANPSI.GT..0) KQ=1
      IF(TANPSI.LT..0) KQ=2
   21 PSIY=ATAN(ABS(TANPSI))
      CALL CORANG(PSIY,KQ,TANG)
      PSIY=TANG
      GO TO 24
C     CHECK FOR NOON TURN CASE WHEN S PRIME EQUALS 90.
   23 IF((ALPHA-ETA)-PI) 25,25,27
```

```
 25     PSIY=3.*PI/2.
        GO TO 24
 27     PSIY=PI/2.
        GO TO 24
 33     IF(COSSPR.GT.0.)PSIY=0.
        IF(COSSPR.LT.0.)PSIY=PI
   24   CONTINUE
        PPSI=PSIY*DEG
C       NOW COMPUTE PADDLE ANGLE PHIP
        SINPHI=-SIN(SPR)*COS(ALPHA-ETA)
        KQ=2
        IF(SINPHI.LE.0.) KQ=3
        PHIP=ATAN(ABS(SINPHI/(1.-SINPHI**2)**.5))
        CALL CORANG(PHIP,KQ,TANG)
        PHIP=TANG
        PPHIP=PHIP*DEG
288     CONTINUE
        CALL GGNG
        IF(IAIR.GT.0) GO TO 2881
        CALL AERO
2881    IF(ISUN.GT.0) GO TO 2882
        CALL SOLAR
2882    XSUM= TA(1)+TS(1)
        YSUM= TA(2)+TS(2)
        ZSUM= TA(3)+TS(3)
        CONS=(GM/(A**3*(1.-E**2)**3))**.5
        CONS=CONS*(1.+E*COS(XNU))**2
        IF(IGRAV.GT.0) GO TO 2883
        CALL GRAV
2883    CONTINUE
        AMAT(1,1)=COS(PSIY)*COS(ETA-ALPHA)
        AMAT(1,2)=-SIN(PSIY)*COS(ETA-ALPHA)
        AMAT(1,3)=SIN(ETA-ALPHA)
        AMAT(2,1)=-SIN(PSIY)
        AMAT(2,2)=-COS(PSIY)
        AMAT(2,3)=0.
        AMAT(3,1)=COS(PSIY)*SIN(ETA-ALPHA)
        AMAT(3,2)=-SIN(PSIY)*SIN(ETA-ALPHA)
        AMAT(3,3)=-COS(ETA-ALPHA)
        DO 253 I=1,3
        TAINT(I,N)=0.
        TSINT(I,N)=0.
        TGINT(I,N)=0.
        DO 253 J=1,3
        TAINT(I,N)=TAINT(I,N)+AMAT(I,J)*TA(J)
        TSINT(I,N)=TSINT(I,N)+AMAT(I,J)*TS(J)
        TGINT(I,N)=TGINT(I,N)+AMAT(I,J)*TG(J)
 253    CONTINUE
        DELTIM=T(N)-T(N-1)
        IF(N.EQ.1) DELTIM=T(N)
        DO 246 I=1,3
        TASUM(I)=TASUM(I)+TAINT(I,N)*DELTIM
        TSSUM(I)=TSSUM(I)+TSINT(I,N)*DELTIM
        TGSUM(I)=TGSUM(I)+TGINT(I,N)*DELTIM
 246    CONTINUE
        GO TO (301,432),ITORIA
 432    GO TO (433,100),IPRINT
  433   WRITE (6,434) N
  434   FORMAT(1H0///1H0,30X,37HORBIT VARIABLES FOR INTERVAL NUMBER  I3)
        TIMIN=T(N)/60.
        WRITE(6,435) TIMIN,T(N)
```

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```
 435 FORMAT(1H0/1H05HTIME=E16.8,9H MINUTES(E16.8,9H SECONDS))
     GO TO 436
301  GO TO (302,100),IPRINT
302  WRITE(5,62) N,TMIN,T(N)
62   FORMAT(1H1,30X,34HORBIT VARIABLES FOR TIME INTERVAL I2,/1H0,20X,5H
    1TIME=E15.8,9H MINUTES(E16.8,9H SECONDS))
 436 WRITE(5,63)DAMEAN,AMEAN
63   FORMAT(1H013HMEAN ANOMALY=E16.8,9H DEGREES(E16.8,9H RADIANS))
     WRITE(5,64)DE2,E2
64   FORMAT(1H018HECCENTRIC ANOMALY=E16.8,9H DEGREES(E16.8,9H RADIANS))
     WRITE(5,68)DXNU,XNU
68   FORMAT(1H013HTRUE ANOMALY=E16.8,9H DEGREES(E16.8,9H RADIANS))
     WRITE(5,202)
202  FORMAT(1H050HDIST IN M,VEL IN M/SEC OR DIST IN FT,VEL IN FT/SEC)
     WRITE(5,41)R,ALT,VEL,VRAD,VPERP
41   FORMAT(1H017HR,H,V,VRAD,VPERP=5(E16.8))
     WRITE(5,42)PGAM,PETA
 42  FORMAT(1H08HGAM,ETA=2(E16.8))
     WRITE(5,54)PALPHA
  54 FORMAT(1H0//1H06HALPHA=E16.8)
     WRITE(5,56)PPSI
 56  FORMAT(1H04HPSI=E16.8)
     WRITE(5,57)PPHIP
 57  FORMAT(1H05HPHIP=E16.8)
     WRITE(5,101) CONS
 101 FORMAT(1H016HANGULAR VELOCITYE16.8)
     WRITE(5,245) TG(1),TG(2),TG(3)
245  FORMAT(1H019HGRAV GRAD BODY COOR3E16.8)
     WRITE(5,240) TA
     WRITE(5,241) TS
241  FORMAT(1H05HSOLAR3E16.8)
     WRITE(5,244) TGINT(1,N),TGINT(2,N),TGINT(3,N)
244  FORMAT(1H018HGRAV GRAD INERTIAL3E16.8)
     WRITE(5,242) TAINT(1,N),TAINT(2,N),TAINT(3,N)
242  FORMAT(1H013HAERO INERTIAL3E16.8)
     WRITE(5,243) TSINT(1,N),TSINT(2,N),TSINT(3,N)
243  FORMAT(1H014HSOLAR INERTIAL 3E16.8)
     WRITE(5,280)DELTIM
280  FORMAT(1H07HDELTIM=E16.8)
     WRITE(5,281)TASUM,TSSUM,TGSUM
281  FORMAT(1H03E16.8)
 100 CONTINUE
     WRITE(5,7676)NODAYS
     WRITE(5,275) TGSUM
275  FORMAT(1H08HGRAV IMP3E16.8)
     WRITE(5,276) TSSUM
276  FORMAT(1H07HSOL IMP3E16.8)
     WRITE(5,277) TASUM
277  FORMAT(1H08HAERO IMP3E16.8)
     XSUM= TGSUM(1)+TASUM(1)+TSSUM(1)
     ZSUM= TGSUM(3)+TASUM(3)+TSSUM(3)
     YSUM= TGSUM(2)+TASUM(2)+TSSUM(2)
     WRITE(5,7060) XSUM,YSUM,ZSUM
7060 FORMAT(1H0,5HXSUM=E12.4,4X,5HYSUM=E12.4,4X,5HZSUM=E12.4)
     XSUM = ABS(XSUM) + ABS(ZSUM)
     YSUM=ABS(YSUM)
     IF(PSPR-90.)8000,8000,8001
8001 PSPR=180.-PSPR
8000 X=(PSPR/5.)+1.
     I=X
     FI=I
     GASX =  THRSTX(I)
```

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```
      GASY =  THRSTY(1)
      IF(X-FI-.1)7000,7000,7001
C     INTERPOLATE
7001  J=I+1
      Y=(FI-1.)*5.
      Z=Y-PSPR
      GASX = GASX+Z*(THRSTX(I)-THRSTX(J))/5.
      GASY = GASY+Z*(THRSTY(I)-THRSTY(J))/5.
7000  CONTINUE
      GASX= GASX* XSUM
      GASY= GASY* YSUM
      SGAST = GASX+GASY
      WRITE(5,7062)SGAST
7062  FORMAT(1H 23HTOTAL GAS THIS ORBIT =  E12.4)
7002  SGASX=SGASX+GASX*FNORB
      SGASY=SGASY+GASY *FNORB
      SGAST=SGASX+SGASY
      WRITE(5,7003)SGAST
7003  FORMAT(1H0,21HTOTAL GAS THIS FAR =   E20.5)
 1000 CONTINUE
      GO TO 7777
      STOP
      END

      SUBROUTINE  SOLAR
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IUV,IUP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANI(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),JPEP(6),TORO(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
     7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA ,NEGO
      DO 20 I=18,40
      DO 20 J=1,3
20    TORO(I,J)=0.0
      TXS = 0.0
      TYS = 0.0
      TZS = 0.0
      INDIC=1
      IF(ABS(PHI)-.00001)200,200,211
200   GO TO (22,23,23,22),IUP
22    INDIC = 2
      GO TO 211
23    INDIC = 3
211   CONTINUE
      GO TO(1,2,1,2),IQP
1     ZI=-1.
      GO TO 3
2     ZI=1.
3     GO TO(4,4,5,5),IUP
4     SZ= -1.
      GO TO 6
5     SZ= +1.
6     CONTINUE
      GO TO (7,8,8,7),IUP
7     SY = -1.
      YC = FL
      GO TO 9
8     SY = 1.
      YC = -(C-FL)
```

```
C       COMPUTE TORQ  ON  YFACE
C
9       FX=0.
        FY=   (1.+AY)*COS(PHI)*COS(PHI)*A*B* SY
        FZ=   (1.-AY)*SIN(ABS(PHI))*COS(PHI)* SZ*A*B
        X=0.
        Z=0.
        CALL TQ(X,YC,Z,18)
C       FOR PADDLES
        TORQ(25,1)=2.*SZ*(-YG)*SIN(ABS(PHI))*H*E*(1.+AP)
C
C       COMPUTE FOR Z-FACE
        FY =    (1.-AZ)*SIN(ABS(PHI))* COS(PHI)* SY *A*C
        FZ =    (1.+AZ)*SIN(PHI)*SIN(PHI)* SZ *A*C
        Z = -SZ*B/2.
        Y = C/2.
        CALL TQ(X,Y,Z,19)
C
C       COMPUTE  FOR  BOOMS
        SPHI = SIN(ABS(PHI))
        CPHI = COS(PHI)
        GO TO (24,25,26),INDIC
24      CONTINUE
        FZ =     (1.+AB5/3.) *(SPHI**2)* B5(4)* SZ
        FY =     (1.-AB5/3.) *(SPHI)*CPHI*B5(4)* SY
        CALL SETSOL(B5,X,Y,Z)
        CALL TQ (X,Y,Z,20)
        FZ =     (1.+AB6/3.)*(SPHI**2)* B6(4)* SZ
        FY =     (1.-AB6/3.)* SPHI * CPHI *B6(4)* SY
        CALL SETSOL(B6,X,Y,Z)
        CALL TQ(X,Y,Z,21)
C
C       COMPUTE  FOR SPHERE  ON B6
C       SL
25      CONTINUE
        FX=0.
        FZ=S6(4)*SZ*CPHI
        FY=S6(4)*SY*SPHI
        CALL SETSOL(S6,X,Y,Z)
        CALL TQ(X,Y,Z,22)
        GO TO (26,26,27),INDIC
C
C       COMPUTE FOR TORUS
26      CONTINUE
        T = 0.
        IF(C5(4))19990,1992,19990
19990   T = C5(4)/C5(5)
        T = ATAN(T/(C5(5)-T))
        T = T*180./PI
        CSA=1.0
        IF(NEGD.GT.0)GO TO 1992
        PHID = ABS(PHI)*180.0/PI
        IF (PHID-T)1990,1991,1991
1990    CSB = (1./T)*PHID+1.
        GO TO 1992
1991    CSA = 2.+(PI-2.)*SPHI
1992    FORCE = (1.-AC5/9.)*CSA*C5(4)
        FZ=SZ*FORCE*SPHI
        FY=SY*FORCE*CPHI
        FX=0.
        CALL SETSOL(C5,X,Y,Z)
        CALL TQ(X,Y,Z,23)
```

```
C
C     COMPUTE  FOR TOP OF BOXS - FSX
      CD = FSX(S)*FSY(S)
      FY = CJ*    (1.-AFS)* SY* SIN(ABS(PHI))*COS(PHI)
      FZ = CJ*    (1.+AFS)*SZ*SIN(PHI)* SIN(PHI)
      CALL SETSOL (FSX,X,Y,Z)
      CALL  TQ(X,Y,Z,24)
C
C   OPEP
27    CONTINUE
      FZ = SZ*(OP(1)*(SPHI**2)*(1.+AOP)+SPHI*CPHI*(1.-AOP)*(OP(2)
     1     *SIN(ABS(PSI))+OP(3)*COS(PSI)))
      TORQ(25,1)=2.*(OPEP(2)-YG)*FZ
C
C    COMPUT E FOR LONG BOOM
      FORCE=BX(4)*(1.+AX/3.)
      FY=CPHI*FORCE*SY
      FZ = SPHI*SZ*FORCE
      CALL SETSOL(BX,X,Y,Z)
      CALL TQ(X,Y,Z,27)
      DO 21 I=18,40
      TXS=TORQ(I,1)           +TXS
      TYS=TORQ(I,2)           +TYS
      TZS = TORQ(I,3)           +TZS
21    CONTINUE
      TXS = TXS*FACTOR*V/1728.
      TYS = TYS*FACTOR*V/1728.
      TZS = TZS*FACTOR*V/1728.
      RETURN
      END
      SUBROUTINE CORANG(ANGD,JACK,ANGR)
      PI=3.1415926
      GO TO(1,2,3,4),JACK
1     ANGR=ANGD
      RETURN
2     ANGR=PI-ANGD
      RETURN
3     ANGR=PI+ANGD
      RETURN
4     ANGR=2.*PI-ANGD
      RETURN
      END
      SUBROUTINE KEPLER(ANMEAN,ECCEN,ERRK,ANECCN,KOUNT)
    1 EG=ANMEAN
      KOUNT=0
    8 SMG=EG-ECCEN*SIN(EG)
      DELE=(ANMEAN-SMG)/(1.-ECCEN*COS(EG))
      IF(ABS(ANMEAN-SMG)-ERRK)4,4,5
    5 IF (KOUNT-25)6,6,7
    6 KOUNT=KOUNT+1
      EG=EG+DELE
      GO TO 8
    4 ANECCN=EG
      RETURN
C     THIS SECTION FOR NON-CONVERGENCE PRINTOUT
    7 WRITE(6,9) KOUNT,ANMEAN,EG,SMG
    9 FORMAT(1H138HKEPLER PROCESS NOT CONVERGING PROPERLY/1H024HNUMBER O
     1F ITERATIONS     I2/1H013HMEAN ANOMALY=E16.8,27HLAST VALUE OF ECCN
     2ANOMALY=E16.8/1H032HLAST CALC VALUE OF MEAN ANOMALY=E16.8)
      RETURN
      END
```

```
      SUBROUTINE GRAV
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IUV,IUP,XG,YG,ZG
     1,H5,H6,H7,COMP,AEL,BEL,H,EEL,W,FL,CEL,PADW,XX,TPSI,TPHI,PAD,ZI,SX
     2,SY,FX,FY,FZ,TA(3)
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),OPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15), VEL,TS(3),UP(3),ALT
     7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA ,NEGO ,NDAYS
     8,TG,CONS,GTHX,GTHY,GTHZ,XXI,YYI,ZZI,XSUM,YSUM,ZSUM
      DIMENSION TG(3)
      RAD=0.017453293
      CONS=CONS*CONS
      IF(XSUM.GT.0.)GO TO 1
      XERR=-0.4
      GO TO 2
    1 XERR=+0.4
    2 IF(YSUM.GT.0.)GO TO 3
      YERR=-0.4
      GO TO 4
    3 YERR=+0.4
    4 IF(ZSUM.GT.0.)GO TO 5
      ZERR=-1.0
      GO TO 6
    5 ZERR=+1.0
    6 TG(1)=2.0*CONS*(YYI-ZZI)*SIN(2.*(GTHX+XERR)*RAD)
      TG(2)=1.5*CONS*(XXI-ZZI)*SIN(2.*(GTHY+YERR)*RAD)
      TG(3)=0.5*CONS*(YYI-XXI)*SIN(2.*(GTHZ+ZERR)*RAD+2.*PSIY)
      RETURN
      END
      SUBROUTINE PNBRA(THETAI,THETA1,THETA2,ECT,IMP,RADFAC)
      PI=3.1415926
      IF(THETA1.LE.THETAI.AND.THETAI.LE.THETA2)GO TO 30
      GO TO 17
   30 B=(THETAI-THETA1)/(THETA2-THETA1)
      S=(1.-2.*B)
      IF(S)13,12,12
   13 SIGN=-1.
      GO TO 14
   12 SIGN=1.
   14 CONTINUE
      IF(ABS(SIN(S)-1.).GT..000001) GO TO 6
    2 RADFAC=1.
      GO TO 10
    6 CONTINUE
      ARCS=ATAN((S*S/(1.-S*S))**.5)
      A1=(2.-4.*B)*(B-B**2)**.5/3.1416
      A2=SIGN*ARCS/PI
      A3=.5
      RADFAC=A1+A2+A3
   10 CONTINUE
      IF(IMP.EQ.1)GO TO 11
      IF(IMP.EQ.2)RADFAC=1.-RADFAC
   11 CONTINUE
      RETURN
C SPECIAL PRINTOUT FOR ERRORS
   17 WRITE(6,18)THETAI,THETA1,THETA2
   18 FORMAT(1H025HERROR IN SUBROUTINE PNBRA/1H021HTHETAI,THETA1,THETA2=
     13(E16.3))
      RETURN
      END
```

```
      SUBROUTINE QUADCK(SN,CS,TN,NQ)
      IF(TN)20,21,22
   21 IF(CS)25,26,26
   20 IF(CS)23,23,24
   22 IF(SN)25,25,26
   26 NQ=1
      GO TO 30
   23 NQ=2
      GO TO 30
   25 NQ=3
      GO TO 30
   24 NQ=4
   30 CONTINUE
C     ANGLE=0,360,NQ=1. ANGLE=90,NQ=2. ANGLE=180,NQ=3. ANGLE=270,NQ=4
C     NQ IS QUADRANT IN WHICH ANGLE LIES. IF ANGLE LIES ON AXIS IT IS
C     ASSIGNED ARBITRARILY AS FOLLOWS.
      RETURN
      END
      SUBROUTINE  SETSOL(P,X,Y,Z)
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IQV,IQP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,FAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),OPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      DIMENSION P(6)
      X=P(1)
      Y=P(2)
      Z=P(3)
      RETURN
      END
      SUBROUTINE TQ(X,Y,Z,N)
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IQV,IQP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,FAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),OPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      X=X-XG
      Y=Y-YG
      Z=Z-ZG
      TORQ(N,1)= Y*FZ -Z*FY
      TORQ(N,2)= Z*FX -X*FZ
      TORQ(N,3)= X*FY -Y*FX
      RETURN
      END
      SUBROUTINE OBJ(X,Y,N)
      DIMENSION X(6),Y(10,4)
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IQV,IQP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,FAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),OPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      IF (COMP- ABS(X(3)))1,2,2
    1 Y(N,1)=X(1)
      Y(N,2)=X(2)
      Y(N,3)=X(3)
```

```
      Y(N,4)=X(4)
      GO TO 998
2     T = ABS(X(2))
      TE = X(6)+X(5)
      IF(H7-H5)40,40,41
41    H5=H7
      H6=0.
      GO TO 42
40    IF(H7.GE.H6)  H6=0.0
42    IF(H5.LE.X(6))GO TO 998
      IF(H5.GT.TE)H5=TE
      IF(H6.LT.X(6))H6=X(6)
      IF(H5-H6-X(5)/2.0)998,30,30
30    Y(N,4)=0
998   CONTINUE
      RETURN
      END
      SUBROUTINE   SHADO
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IUV,IUP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),BS(6),S2(6),C3(6)
     4,C1(6),CS(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),JPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,ACS,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
     7,AP,GAMA,AX,FACTOR,NOSHAD,GAMMA
      ZP(PHI)=(Z8-H/2.)*COS(PHI)
      YP(PHI)= (H/2.-Z8)* SIN(PHI)
900   FORMAT (1H ,31HH1,H2,H3,H4,COMP,...BODYXPADDLE    ,5E20.8)
901   FORMAT (1H 29HH1,H2,COMP,....PADDLE BY BODY, 3E20.4 )
      TPSI = SIN(ABS(PSI))/COS(PSI)
      TPHI = SIN(ABS(PHI))/COS(PHI)
      CYL(5,1)=COPEP(1)
      CYL(5,2)=COPEP(2)
      CYL(5,3)=COPEP(3)
      CYL(5,4)=COPEP(4)
      BOOM(1,1)= B6(1)
      BOOM(1,2)= B6(2)
      BOOM(1,3)= B6(3)
      BOOM(1,4)= B6(4)
      SPH (1,1)= S6(1)
      SPH (1,2)= S6(2)
      SPH (1,3)= S6(3)
      SPH (1,4)= S6(4)
      PLNE(7,1) =F4Y(1)
      PLNE(7,2) =F4Y(2)
      PLNE(7,3) =F4Y(3)
      PLNE(7,4) =F4Y(4)
      PLNE(8,1) =F4X(1)
      PLNE(8,2) =F4X(2)
      PLNE(8,3) =F4X(3)
      PLNE(8,4) =F4X(4)
      PLNE(9,1) =F5Y(1)
      PLNE(9,2) =F5Y(2)
      PLNE(9,3) =F5Y(3)
      PLNE(9,4) = F5Y(4)
      PLNE(10,1)=F5X(1)
      PLNE(10,2)=F5X(2)
      PLNE(10,3)=F5X(3)
      PLNE(10,4)=F5X(4)
      CYL(3,1)= CANT(1)
      CYL(3,2)= CANT(2)
```

BAARINC

```
      CYL(3,3)= CANT(3)
      CYL(3,4)= CANT(4)
      BOOM(2,1)= B5(1)
      BOOM(2,2)= B5(2)
      BOOM(2,3)= B5(3)
      BOOM(2,4)= B5(4)
      SPH (2,1)= S2(1)
      SPH (2,2)= S2(2)
      SPH (2,3)= S2(3)
      SPH (2,4)= S2(4)
      CYL (1,1)= C3(1)
      CYL (1,2)= C3(2)
      CYL (1,3)= C3(3)
      CYL (1,4)= C3(4)
      CYL (2,1)= C1(1)
      CYL (2,2)= C1(2)
      CYL (2,3)= C1(3)
      CYL (2,4)= C1(4)
      CYL(4,1) =C5(1)
      CYL(4,2) = C5(2)
      CYL(4,3) =C5(3)
      CYL(4,4) =C5(4)
      BOOM(3,1)=BX(1)
      BOOM(3,2)=BX(2)
      BOOM(3,3)=BX(3)
      BOOM(3,4)=BX(4)
      A8=H*E
      A5=C*B
      Y5=C/2.0
      Z5=B/2.0
       X8=E/2.0
      Z8=H/2.0
       HE=H*E
       AB=A*B
      PI2=PI/2.
      TE = ABS(ABS(PSI)-PI2)
      IF(TE-.0001)2001,2001,2005
C     PSI=90
C
2001   A5=0.
      GO TO (2050,2050,2051,2051),IQV
2050  SPH(1,4)=0.
      CYL(3,4)=0.
      GO TO 2052
2051  CYL(4,4)=0.0
      PLNE(10,4)=0.0
      CYL(5,4)=0.0
2052  CONTINUE
       IF(ABS(PHI)-PI 2 )80,2003,80
2003  A8=0.
       HE=0.
       GO TO 80
C      PSI NOT 90
2005   IF(ABS(PHI)-PI 2 )2006,2007,2006
2007   A8=0.
       HE=0.
2006  GO TO (20061,20062),NOSHAD
20062 IF(PSI)80,2000,80
20061 IF(PSI)1,2000,3
2000   BOOM(3,4)=0.
       A8=0.
```

BAARINC

```
      AB=0.
      HE=0.
      GO TO 80
1     BL = FL
2     GO TO (4,5,4,5),IUP
3     BL = C-FL
      GO TO (5,4,5,4),IUP
4     ZI =-1
      GO TO 6
5     ZI = 1
6     GO TO (7,8,8,7),IQV
7     XI = 1
      H5=0
      IF(ABS(PHI)-PI2  )112,111,112
111   H5=.5*H/TPSI
112   H6=(C-BL)/TPSI-W-E
      IF(H6.GT.H5) GO TO 113
      H6=H5
113    IF(H6.LE.0.)GO TO 90
C     THERE IS A SHADOW
      BOOM(3,1)= BX(1)-.5*H6
      BOOM(3,4)= (BX(5)-H6)*BX(4)/BX(5)
       GO TO 90
8     XI = -1
90    PADW=A/2.+W+E
      PAD= A/2.+W
      COMP = (H/2.)*COS(PHI)
      GO TO (10,10,11,11),IQV
C
C     NEG END SHADOED
10    CALL SETUP(B6)
      CALL FLIM(B6,BOOM,1)
C     SHADO  EP4
C
      CALL SETUP(F4X)
      CALL OBJ(F4X,PLNE,8)
      IF(PLNE(8,4)-F4X(4))400,401,400
400   PLNE(7,4)=0
      GO TO 402
401   PLNE(7,4)=F4Y(4)
402   CONTINUE
C
C     SHADO S6
      CALL SETUP(S6)
      CALL OBJ(S6,SPH,1)
C
C     SHADO CANT
      CALL SETUP(CANT)
      CALL OBJ(CANT,CYL,1)
      GO TO 12
C
C     POS END SHADOED
C     BOOM 5 SHADO
11    CALL SETUP(B5)
      CALL FLIM(B5,BOOM,2)
C
C     OPEP  CYLINDER  SHADO
      T = COPEP(2)-FL
      IF((H5.GT.T).AND.(T.GT.H6).OR.(H7.GT.T)) GO TO 1000
      GO TO 1001
1000  CYL(5,4)=0
1001  CONTINUE
```

```
C       CYL 5
C
        CALL SETUP(C5)
        CALL OBJ(C5,CYL,4)
        CALL SETUP(F5X)
        CALL OBJ(F5X,PLNE,10)
        IF(PLNE(10,4)-F5X(4))403,404,403
403     PLNE(9,4) = 0.
        GO TO 405
404     PLNE(9,4) =F5Y(4)
405     CONTINUE
C       SPH2
C
        CALL SETUP(S2)
        CALL OBJ(S2,SPH,2)
C
C       EP3
        CALL SETUP(C3)
        CALL OBJ(C3,CYL,1)
C
C       EP1
        CALL SETUP(C1)
        CALL OBJ(C1,CYL,2)
C       COMPUTE SAME FOR BODY
C
C       NOW TO COMPUTE SHADING ON BODY AND PADDLES VIA STL
C
12      CONTINUE
        COMP = H*COS(PHI)
        Q = C-BL
        WEPSI =(W+E)*TPSI
        WPSI  =W* TPSI
        H2PHI=(H/2.)*SIN(ABS(PHI))
        A1=B*C
        Y1=C/2.
        Z1=B/2.
201      IF(ABS(PHI)-PI 2 )21,80,21
21      IF(COMP.LT.B) GO TO 30
        TE =.5*B*TPHI
        H1 = WEPSI + TE + Q
        H2 = WEPSI -TE + Q
        H3 = WPSI +TE + Q
        H4 = WPSI -TE + Q
        IF((H1.LE.C).AND.(H2.LE.C).AND.(H3.LE.C).AND.(H4.LE.C)) GO TO 22
C       FOR  CASE 1-B
        IF((H1.GT.C).AND.(H2.LE.C).AND.(H3.LE.C).AND.(H4.LE.C))GO TO 23
C       1-C
        IF((H1.GT.C).AND.(H2.GT.C).AND.(H3.LE.C).AND.(H4.LE.C))GO TO 24
C       1-D
        IF((H1.GT.C).AND.(H3.GT.C).AND.(H2.LE.C).AND.(H4.LE.C))GO TO 25
C       1-E
        IF((H1.GT.C).AND.(H2.GT.C).AND.(H3.GT.C).AND.(H4.LE.C))GO TO 26
C       1-F
        IF((H1.GE.C).AND.(H2.GE.C).AND.(H3.GE.C).AND.(H4.GE.C)) GO TO 27
        WRITE (6,900)H1,H2,H3,H4,COMP
        GO TO 40
C-      1-A
22      A2=B*(H1-H3)
        Y2=(H1+H4)/2.
        Z2=B/2.
        AS=A1-A2
```

```
      Y5=(A1*Y1 -A2*Y2)/A5
      Z5=(A1*Z1 -A2*Z2)/A5
       GO TO 40
23    A2=B*(H1-H3)
      Y2=.5*(H1+H4)
      Z2=.5*B
      A3=(.5*B*(H1-C)**2)/(H1-H2)
      Y3= (2.*C+H1)/3.
      Z3= B*(2.*H1-3.0*H2+C)/(3.0*(H1-H2))
      A5=A1-A2+A3
      Y5=(A1*Y1-A2*Y2+ A3*Y3)/A5
      Z5=(A1*Z1-A2*Z2+ A3*Z3)/A5
      GO TO 40
24    A2=(B*(H3-H4))/2.0
      Y2=(2.0*H3+H4)/3.0
      Z2=B/3.0
      A3=B*(C-H3)
      Y3=(C+H3)/2.0
      Z3=B/2.0
      A5=A1-A2-A3
      Y5=(A1*Y1-A2*Y2-A3*Y3)/A5
      Z5=(A1*Z1-A2*Z2-A3*Z3)/A5
      GO TO 40
25    A2 =(.5*B*(C-H4)**2)/(H3-H4)
      Y2= (2.0*C+H4)/3.0
      Z2=(B*(C-H4))/(3.0*(H3-H4))
      A3=.5*B*((C-H2)**2)/(H1-H2)
      Y3=(2.0*C +H2)/3.0
      Z3=(B*(C-H2))/(3.0*(H1-H2))
      A5=A1-A2+A3
      Y5=(A1*Y1-A2*Y2+A3*Y3)/(A1-A2+A3)
      Z5=(A1*Z1-A2*Z2+A3*Z3)/A5
      GO TO 40
26    A2=.5*B*((C-H4)**2)/(H3-H4)
      Y2= (2.0*C +H4)/3.0
      Z2= B*(C-H4)/(3.0*(H3-H4))
      A5=A1-A2
      Y5=(A1*Y1-A2*Y2)/A5
      Z5=(A1*Z1-A2*Z2)/A5
      GO TO 40
27    A5=B*C
      Y5=Y1
      Z5=Z1
      GO TO 40
C     HCOS(PHI)  LESS THAN B
C
30    H1 = WEPSI + H2PHI + Q
      H2 = WEPSI - H2PHI  + Q
      H3 = WPSI +H2PHI +Q
      H4 = WPSI  - H2PHI +Q
        HCOS = H*COS(PHI)
      IF((H1.LE.C).AND.(H2.LE.C).AND.(H3.LE.C).AND.(H4.LE.C)) GO TO 32
      IF((H1.GT.C).AND.(H2.LE.C).AND.(H3.LE.C).AND.(H4.LE.C)) GO TO 33

      IF((H1.GT.C).AND.(H2.GT.C).AND.(H3.LE.C).AND.(H4.LE.C)) GO TO 34
      IF((H1.GT.C).AND.(H2.GT.C).AND.(H3.GT.C).AND.(H4.LE.C)) GO TO 36
      IF((H1.GE.C).AND.(H2.GE.C).AND.(H3.GE.C).AND.(H4.GE.C)) GO TO 40
      IF((H1.GT.C).AND.(H3.GT.C).AND.(H2.LE.C).AND.(H4.LE.C)) GO TO 35
      WRITE(6,900) H1,H2,H3,H4,COMP
C     2-A
32    A2=HCOS*(H1-H3)
      Y2=(H1+H4)/2.0
```

```
      Z2=Z1
      A5= A1-A2
      Y5= (A1*Y1-A2*Y2)/A5
      Z5= (A1*Z1-A2*Z2)/A5
      GO TO 40
C     2-B
33    A2=HCOS*(H1-H3)
      Y2=(H1+H4)/2.0
      Z2=Z1
      A3=.5*HCOS*((H1-C)**2)/(H1-H2)
      Y3= (2.0*C + H1)/3.0
      Z3= ((2.0*C+H1-3.0*H2)*HCOS/(6.0*(H1-H2)))+ B/2.0
      A5=A1-A2+A3
      Y5=(A1*Y1-A2*Y2+A3*Y3)/A5
      Z5=(A1*Z1-A2*Z2+A3*Z3)/A5
      GO TO 40
C     2-C
34    A2= .5*HCOS*(H3-H4)
      Y2=(2.0*H3+H4)/3.0
      Z2=(3.0*B-HCOS)/6.0
      A3=HCOS*(C-H3)
      Y3= .5*(C+H3)
      Z3=Z1
      A5=A1-A2-A3
      Y5=(A1*Y1-A2*Y2-A3*Y3)/A5
      Z5=(A1*Z1-A2*Z2-A3*Z3)/A5
      GO TO 40
C     2-D
35    A2=.5*HCOS*((C-H4)**2)/(H3-H4)
      Y2= (2.0*C +H4)/3.0
      Z2= ((2.0*C+H4-3.0*H3)*HCOS/(6.0*(H3-H4)) + B/2.0)
      A3=.5*HCOS*((C-H2)**2)/(H1-H2)
      Y3= (2.0*C +H2)/3.0
      Z3= ((2.0*C +H2-3.0*H1)*HCOS/(6.0*(H1-H2)) + B/2.0)
      A5= A1-A2+A3
      Y5 =(A1*Y1 -A2*Y2-A3*Y3)/A5
      Z5 =(A1*Z1 -A2*Z2-A3*Z3)/A5
      GO TO 40
C     2-E
36    A2=.5*HCOS*((C-H4)**2)/(H3-H4)
      Y2=(2.0*C + H4)/3.0
      Z2=((2.0*C +H4-3.0*H3)*HCOS/(6.0*(H3-H4)) + B/2.0)
      A5=A1-A2
      Y5= (A1*Y1 - A2*Y2)/A5
      Z5= (A1*Z1 - A2*Z2)/A5
      GO TO 40
C     2-F
C
C     SHADING OF PADDLE BY BODY
C
40     A1=HE
      Z1 = .5*H
      X1 = .5*E
      BSEC= B/COS(PHI)
      CPSI= 1./TPSI
      B2PHI= B*TPHI/2.0
      IF(COMP-B)50,42,42
42    H1 = CPSI*(Q +B2PHI) - W
      H2 = CPSI*(Q -B2PHI) - W
      IF((H1.LE.0.0).AND.(H2.LE.0.0))GO TO 80
      IF((E.GE.H1).AND.(H1.GT.0.0).AND.(H2.LT.0.0)) GO TO 43
```

```
      IF((E.GE.H1).AND.(H2.GE.0.0)) GO TO 44
      IF((H1.GT.E).AND.(E.GE.H2).AND.(H2.GE.0.0)) GO TO 45
      IF((H1.GT.E).AND.(H2.GT.E)) GO TO 46
      IF((H1.GT.E).AND.(H2.LT.0.0))GO TO 455
      WRITE (6,901) H1,H2,COMP
      GO TO 80
455   H1=E
      GO TO 43
43    A2 =(H1**2)*BSEC/(2.0*(H1-H2))
      Z2 =(H1-3.0*H2)*BSEC/(6.0*(H1-H2)) + H/2.0
      X2 = H1/3.0
      A8 = A1-A2
      Z8 =(A1*Z1 -A2*Z2)/A8
      X8 =(A1*X1 -A2*X2)/A8
      GO TO 80
44    A2 = .5*(H1+H2)*BSEC
      Z2 =  (H1-H2)*BSEC/(6.0*(H1+H2)) + H/2.0
      X2 = (H1**2+H1*H2+H2**2)/(3.0*(H1+H2))
      A8 = A1-A2
      Z8 =(A1*Z1-A2*Z2)/A8
      X8 =(A1*X1-A2*X2)/A8
      GO TO 80
45    A2 = (H1+H2)*BSEC*.5
      Z2 =((H1-H2)*BSEC)/(6.0*(H1+H2)) + H/2.0
      X2 = (H1**2 + H1*H2 + H2**2)/(3.0*(H1+H2))
      A3 = ((H1-E)**2)*BSEC/(2.0*(H1-H2)).
      Z3 = BSEC*(2.0*E + H1-3.0*H2)/(6.0*(H1-H2)) + H/2.0
      X3 =(H1 + 2.0*E)/3.0
      A8 = A1-A2+A3
      Z8 = (A1*Z1 -A2*Z2+A3*Z3)/A8
      X8 =(A1*X1 - A2*X2+A3*X3)/A8
      GO TO 80
46    A2=BSEC*E
      A8=A1-A2
      Z8=Z1
      X8=X1
      GO TO 80
50    H1 = CPSI*(Q+.5*H*SIN(ABS(PHI))) - W
      H2 = CPSI*(Q-.5*H*SIN(ABS(PHI))) - W
      IF((H1.LE.0.0).AND.(H2.LE.0.0))GO TO 80
      IF ((E.GE.H1).AND.(H1.GT.0.0).AND.(H2.LT.0.0)) GO TO 53
      IF((E.GE.H1).AND.(H2.GE.0.0))GO TO 54
      IF((H1.GT.E).AND.(H2.LT.0.0)) GO TO 55
      IF((H1.GT.E).AND.(E.GE.H2).AND.(H2.GE.0.0)) GO TO 56
      IF((H1.GE.E).AND.(H2.GE.E)) GO TO 58
      WRITE (6,901)H1,H2,COMP
      GO TO 80
53    A2 = H1*H1*H/(2.0*(H1-H2))
      Z2 = H*(2.*H1 - 3.*H2)/(3.0*(H1-H2))
      X2 = H1/3.
      A8 = A1-A2
      X8 =(A1*X1-A2*X2)/A8
      Z8 =(A1*Z1-A2*Z2)/A8
      GO TO 80
54    A2 = H*(H1+H2)*.5
      Z2 = H*(2.*H1+H2)/(3.0*(H1+H2))
      X2 = (H1*H1 +H1*H2 +H2*H2)/(3.0*(H1+H2))
      A8 = A1-A2
      Z8 =(A1*Z1 - A2* Z2)/A8
      X8 =(A1*X1 - A2* X2)/A8
      GO TO 80
55    A2 = H*H1*H1/(2.*(H1-H2))
```

```
      Z2 = H*(2.*H1 - 3.*H2)/(3.0*(H1-H2))
      X2 = H1/3.0
      A3 = ((H1-E)**2)*H/(2.*(H1-H2))
      Z3 = H*(2.*H1 - 3.*H2+E)/(3.0*(H1-H2))
      X3 = (2.*E + H1)/3.0
      A8 = A1-A2+A3
      Z8 = (A1*Z1 -A2*Z2 +A3*Z3)/A8
      X8 = (A1*X1 -A2*X2 +A3*X3)/A8
      GO TO 80
56    A2=H*(H1+H2)/2.0
      Z2=H*(2.0*H1 + H2)/(3.0*(H1+H2))
      X2=(H1*H1 + H1*H2 +H2*H2)/(3.0*(H1+H2))
      A3= H*((H1-E)**2)/(2.0*(H1-H2))
      Z3= H*(2.0*H1 - 3.0*H2 + E)/(3.0*(H1-H2))
      X3= (2.0*E + H1)/3.0
      A8 = A1 -A2 +A3
      Z8 = (A1*Z1 - A2*Z2 + A3*Z3)/A8
      X8 = (A1*X1 - A2*X2 + A3*X3)/A8
      GO TO 80
58    A8 = 0.0
80    CONTINUE
C
C     ENTER  CENTROIDS INTO FLAT
      GO TO (81,82,83,84),IQV
81    PLNE(4,1)= A/2.
      PLNE(4,2)=FL-YS
      PLNE(4,3)=Z5-B/2.
      PLNE(4,4)= A5
      PLNE(3,1)=0.0
      PLNE(3,2)=FL
      PLNE(3,3)=0.0
      PLNE(5,4)=0.0
      PLNE(3,4)=A8
      PLNE(6,4)= 0.0
      PLNE (1,1)=(A+E)/2.0+W
      PLNE (1,2)= 0.0
      PLNE (1,3)=0.0
      PLNE(1,4)=HE
      PLNE(2,1)=-(A/2.+W+X8)
      PLNE(2,2)= YP(PHI)
      PLNE(2,3)= ZP(PHI)
      PLNE(2,4)=A8
      GO TO (85,85,86,86),IQP
86    PLNE(4,3)= -PLNE(4,3)
      PLNE (2,2)= -PLNE(2,2)
      PLNE (2,3) = -PLNE(2,3)
      GO TO 85
82    PLNE(4,4)=0.0
       PLNE(3,4)=A8
      PLNE(3,1)=0.0
      PLNE(3,2)=FL
      PLNE(3,3)=0.0
      PLNE(5,4)=0.0
      PLNE (8,1)=F4X(1)-F4Y(5)
      PLNE(10,1)=F5X(1)-F5Y(5)
      PLNE(6,1)= -(A/2.)
      PLNE(6,2)=  (FL-YS)
      PLNE(6,3)=(Z5 - B/2.0)
      PLNE(6,4)= A5
      PLNE(1,1)= (A/2. + W + X8)
      PLNE(1,2)= YP(PHI)
```

```
      PLNE(1,3)= ZP(PHI)
      PLNE(1,4)= AB
      PLNE(2,4)=HE
      PLNE(2,1)= -((A+E)/2.+W)
      PLNE(2,2)=0.0
      PLNE(2,3)=0.0
      GO TO(85,85,88,88),IQP
88    PLNE(6,3) = - PLNE(6,3)
      PLNE(1,2) = - PLNE(1,2)
      PLNE(1,3)= - PLNE(1,3)
      GO TO 85
83    PLNE(4,4)=0.0
      PLNE(5,4)=AB
      PLNE(5,1)=0.0
      PLNE(5,2)=-(C-FL)
      PLNE(5,3)=0.0
      PLNE(3,4)=0.0
      PLNE(9,2)=F5Y(2)-F5X(5)
      PLNE(7,2)=F4Y(2)-F4X(5)
      PLNE(10,1)=F5X(1)-F5Y(5)
      PLNE (8,1)=F4X(1)-F4Y(5)
      PLNE(6,1)=-A/2.0
      PLNE(6,2)=Y5-C+FL
      PLNE(6,3)= B/2.0 -Z5
      PLNE(6,4)= A5
      PLNE(1,4)= AB
      PLNE(1,1)= A/2.+W+X8
      PLNE(1,2)= -YP(PHI)
      PLNE(1,3)= -ZP(PHI)
      PLNE(2,4)=HE
      PLNE(2,1)= -((A+E)/2. + W)
      PLNE(2,2)= 0.0
      PLNE(2,3)= 0.0
      GO TO (85,85,89,89),IQP
89    PLNE (6,3)= -PLNE(6,3)
      PLNE(1,2)= -PLNE(1,2)
      PLNE (1,3)= -PLNE(1,3)
      GO TO 85
84    PLNE(4,4)= A5
      PLNE(4,1)= A/2.
      PLNE(4,2)=+Y5 -C + FL
      PLNE(4,3)= B/2. - Z5
      PLNE(5,4)=AB
      PLNE(5,1)=0.0
      PLNE(5,2)=-(C-FL)
      PLNE(5,3)=0.0
      PLNE(3,4)=0.0
      PLNE(7,2)=F4Y(2)-F4X(5)
      PLNE(9,2)=F5Y(2)-F5X(5)
      PLNE(6,4)= 0.0
      PLNE(1,4)=HE
      PLNE(1,1)= (A+E)/2. + W
      PLNE(1,2)= 0.0
      PLNE(1,3)= 0.0
      PLNE(2,4)= AB
      PLNE(2,1)= -(A/2.+W + X8)
      PLNE(2,2)= - YP(PHI)
      PLNE(2,3)= - ZP(PHI)
      GO TO (85,85,92,92),IQP
92    PLNE(4,3)= -PLNE(4,3)
      PLNE(2,2)= -PLNE(2,2)
      PLNE(2,3)= -PLNE(2,3)
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85     CONTINJE
100.   CONTINJE
       RETURN
       END

       SUBROUTINE GGNG
       COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IUV,IUP,XG,YG,ZG
      1,H5,H6,H7,COMP
      2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
      3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANI(6),B5(6),S2(6),C3(6)
      4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
      510,4),OPEP(6),TORQ(40,3),BL
      6,AY,AZ,ABS,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA
       PSIV= -PSIY
       AR = PSIV
       INDIC=1
       APSIV = ABS(PSIV)
15     IF(APSIV -.5*PI)1,1,2
1      IQ = 1
       GO TO 10
2      IF(APSIV-PI)3,30,4
30     APSIV=0.0
       GO TO 31
3      APSIV=PI-APSIV
31     IQ=2
       GO TO 10
4      IF(APSIV -1.5* PI) 5,5,6
5      IQ = 3
       APSIV= PI-APSIV
       GO TO 10
6      IF(APSIV - 2.*PI) 7, 7, 100
7      IQ=4
       APSIV = APSIV-2. *PI
10     IF(AR)11,12,12
11     APSIV=-APSIV
       IQ= 5-IQ
12     GO TO (13,14,144),INDIC
13     PSI = APSIV
       IQV = IQ
       AR= PHIP
       APSIV = ABS(PHIP)
       INDIC=2
       GO TO 15
100    CONTINJE
       GO TO 101
14     PHI = APSIV
101    IQP = IQ
       INDIC = 3
       AR = GAMA
       APSIV = ABS(GAMA)
       GO TO 15
144    GAMMA=ABS(APSIV)
       T = PI/2.
       TE = ABS(ABS(PSI)-T)
       IF(TE-.00001)300,300,301
300    IF(PSI)302,303,303
302    PSI=-T
       GO TO 301
303    PSI=T
301    TE=ABS(ABS(PHI)-T)
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```
      IF(TE-.00001)304,304,305
304   IF(PHI)306,307,307
306   PHI=-T
      GO TO 305
307   PHI=T
305   IF(ABS(PSI)-.0001)308,308,309
308   PSI=0.0
309   IF(ABS(PHI)-.0001)310,310,311
310   PHI = 0.0
311   CONTINUE
      RETURN
      END
      SUBROUTINE AERO
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IQV,IQP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CAN1(6),B5(6),S2(6),C3(6)
     4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),OPEP(6),TORQ(40,3),BL
     6,AY,AZ,AB5,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
     7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA ,NEGO
C     AERODYNAMIC  TORQUE
      XFACEY(ARX)=2.*ARX*SGMA*SIN (ABS(PSI))*COS(PSI) *SY
      YFACEX(ARY)=SX*2.0*ARY*SGMA*SIN(ABS(PSI))*COS(PSI)
      YFACEY(ARY)= SY*2.* ARY*(2.-SGMAP)*(SIN(PSI))**2
      XFACEX(ARX)=SX*2.*ARX*(2.-SGMAP)*COS(PSI)**2
998   FORMAT(1H ,7E15.3)
      DO 12 I=1,20
      DO 12 J=1,3
12    TORQ(I,J)=0.0
      GO TO (1,2,2,1),IQV
1     SX = -1.
      GO TO 3
2     SX=1.
3     GO TO (4,4,5,5),IQV
4     SY=-1.
      GO TO 6
5     SY= 1.
6     CONTINUE
      GO TO (60,61,60,61),IQP
60    SZ=-1.
      GO TO 62
61    SZ=1.
62    CONTINUE
      CALL SHADO
      N=3
      DO 7 J=3,10,2
      JJ=J+1
      ARX = PLNE(JJ,4)
      ARY = PLNE(J,4)
      PLNE(J,3)= PLNE(J,3)-ZG
      PLNE(J,1)= PLNE(J,1)-XG
      TORQ(N,1)= -PLNE(J,3)* YFACEY(ARY) - PLNE(JJ,3)*XFACEY(ARX)
      TORQ(N,2)= +PLNE(J,3)* YFACEX(ARY)+ PLNE(JJ,3)* XFACEX(ARX)
      TORQ(N,3)=PLNE(J,1)*YFACEY(ARY)- (PLNE(J,2)-YG)*YFACEX(ARY)+
     1           PLNE(JJ,1)*XFACEY(ARX)-(PLNE(JJ,2)-YG)*XFACEX(ARX)
7     N=N+1
      DO 8 J=1,2
      PLNE(J,3)= PLNE(J,3)-ZG
      PLNE(J,1)= PLNE(J,1)-XG
      FX = 2.*PLNE(J,4)*SGMA*COS(PSI)*COS(PSI)*SIN(ABS(PSI))*SX
      FY=2.*PLNE(J,4)*SY*((2.-SGMA-SGMAP)*(COS(PHI)**3)*(SIN(PSI)**2)+
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     1 SGMA*(SIN(PSI)**2)*COS(PHI))
      FZ=PLNE(J,4)*(2.-SGMA-SGMAP)*SIN(ABS(PHI))*(COS(PHI)**2)*(SIN(ABS
     1(PSI))**2)*SZ*SY
      TORQ(J,1)=(PLNE(J,2)-YG)*FZ - PLNE(J,3)*FY
      TORQ(J,2)= PLNE(J,3)*FX - PLNE(J,1)*FZ
      TORQ(J,3)= PLNE(J,1)*FY - (PLNE(J,2)-YG)*FX
8     CONTINUE
C     TORQ ON LONG BOOM
      AR = BOOM(3,4)
      CD = 2.0*(1.0-(1.0-SGMAP)/3.0)
      FX=CD*AR*SX*SIN(ABS(PSI))*COS(PSI)
      FY =(2.0+2.*(1.-SGMAP)/3.)*SY*AR*(SIN(PSI)**2)
      CALL TORQUE(BOOM,3,8)
C
C     TORQ ON OTHER BOOMS
      N=9
      DO 9 J=1,2
      AR=BOOM(J,4)
      FX= (2.0+2.0*(1.0-SGMAP)/3.0)*SX*AR*(COS(PSI)**2)
      FY=CD*AR*SY*COS(PSI)*SIN(ABS(PSI))
      CALL TORQUE(BOOM,J,N)
9     N=N+1
C
C     TORQUE  ON SPHERES
      DO 10  J=1,2
      AR=SPH(J,4)
      FX=AR*2.0*COS(PSI)*SX
      FY= AR*2.0* SIN(ABS(PSI))*SY
      CALL TORQUE(SPH,J,N)
10    N=N+1
C
C     TORQUE ON CYL.
      DO 11 J=1,2
      AR= CYL(J,4)
      FX=(2.0+2.0*(1.0-SGMAP)/3.0)*SX*AR*(COS(PSI)**2)
      FY=CD*AR*SIN(ABS(PSI))*COS(PSI)*SY
      CALL TORQUE(CYL,J,N)
11    N=N+1
      FX =SX*COS(PSI)* CD * CYL(3,4)
      FY =SX*SIN(ABS(PSI))* CD* CYL(3,4)
      CALL TORQUE(CYL,3,N)
      N=N+1
C
C     OPEP TORQUE
C     OPEP CYL
      CD = (1.0+(1.0-SGMAP)/3.0)*2.0
      FX = SX*COS(PSI)*CD*CYL(5,4)
      FY=SY*SIN(ABS(PSI))*CD*CYL(5,4)
      CALL TORQUE(CYL,5,17)
      TORQ(17,3)=TORQ(17,3)-OPEP(2)*SX*2.0*OPEP(4)*COS(PSI)
      T=0.
      IF(CS(4))19990,1992,19990
19990 T = CS(4)/CS(5)
      T = ATAN(T/(CS(5)-T))
      T = T*180./PI
      IF(NEGO.GT.0)GO TO 1993
      TE = GAMMA
      GAMAD = GAMMA*180./PI
      GO TO 1994
C     FOR PERPENDICULAR LOOP /POGI
1993  TE = -ABS(PSI)+ PI/2.
```

```
      GAMAD= TE*180./PI
1994  IF(GAMAD-T)1990,1991,1991
1990  CSA= (1./T)* GAMAD + 1.0
      GO TO 1992
1991  CSA=2.+(PI-2.)* SIN(TE)
1992  FORCE = 2.*(1.-(1.-SGMAP)/9.) *CYL(4,4)  *CSA
      FX = SX*COS(GAMMA)*FORCE*COS(PSI)
      FY = SY*COS(GAMMA)*FORCE*SIN(ABS(PSI))
      FZ=FORCE*SIN(GAMA)
      CALL TORQUE(CYL,4,16)
      TORQ(15,1)=TORQ(16,1)+(CYL(4,2)-YG)*FZ
      TORQ(15,2) = TORQ(16,2)-(CYL(4,1)-XG)* FZ
      TX=0.
      TY=0.
      TZ=0.
      DO 13 I=1,20
      TX= TX + TORQ(I,1)
      TY= TY + TORQ(I,2)
13    TZ= TZ + TORQ(I,3)
      X = (ALT-100.)/50.+1.
      IF(X-1.)6000,6000,6001
6000  I=1
      GO TO 6010
6001  IF(X-14.)6003,6002,6002
6002  I=14
      GO TO 6010
6003  I=X
      FI=I
      IF(X-FI-.0001)6010,6010,6004
C
C  INTERPOLATIO N REQUIRED
6004  Z = 100.+(FI-1.)*50.0
      DENS = (ATMO(I+1)-ATMO(I))*(ALT-Z)/50.+ATMO(I)
      GO TO 6011
6010  DENS = ATMO(I)
6011  DENS=(DENS*(VEL**2)*.5)/1728.
      TX = DENS*TX
      TY=DENS*TY
      TZ = DENS*TZ
      RETURN
      END
      SUBROUTINE TORQUE (BO,J,N)
C TORQUE
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IOV,IOP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
     3,B6(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANI(6),BS(6),S2(6),C3(6)
     4,C1(6),C5(6),6X(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
     510,4),DPEP(6),TORQ(40,3),BL
     6,AY,AZ,ABS,AB6,AF5,AOP,AC5,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      DIMENSION BO(10,4)
      TORQ(N,1)= -(BO(J,3)-ZG)*FY
      TORQ(N,2)=(BO(J,3)-ZG)*FX
      TORQ(N,3)=(BO(J,1)-XG)*FY -(BO(J,2)-YG)*FX
      RETURN
      END
      SUBROUTINE SETUP(AA)
C     SETUP
      DIMENSION AA(6)
      COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IOV,IOP,XG,YG,ZG
     1,H5,H6,H7,COMP
     2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
```

```
      3,B5(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
      4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
      510,4),OPEP(6),TORQ(40,3),BL
      5,AY,AZ,ABS,AB6,AF5,AOP,ACS,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
       H5=(PA)W-XI*AA(1))*TPSI-BL+ZI*AA(3)*TPHI
       H6=(PA)-XI*AA(1))*TPSI-BL+ZI*AA(3)*TPHI
       H7=(A/2.-XI*AA(1))*TPSI
       IF(H7)1,2,2
1      WRITE (6,60)H7,H5,H6,XI,ZI,AA(1),AA(3)
60     FORMAT(1H0 18HH7,H5,H6,XI,ZI,X,Z    7E12.5)
2      CONTINUE
       RETURN
       END
       SUBROUTINE FLIM(X,Y,N)
       COMMON PSI,PSIV,PSIY,PI,PHI,PHIP,IOV,IOP,XG,YG,ZG
      1,H5,H6,H7,COMP
      2,A,B,H,E,W,FL,C,PADW,XI,TPSI,TPHI,PAD,ZI,SX,SY,FX,FY,FZ,TX,TY,TZ
      3,B5(6),S6(6),F4Y(6),F4X(6),F5X(6),F5Y(6),CANT(6),B5(6),S2(6),C3(6)
      4,C1(6),C5(6),BX(6),COPEP(6),BOOM(10,4),SPH(10,4),CYL(10,4),PLNE(
      510,4),OPEP(6),TORQ(40,3),BL
      6,AY,AZ,ABS,AB6,AF5,AOP,ACS,V,ATMO(15),VEL,TXS,TYS,TZS,OP(3) ,ALT
      7,AP,GAMA,AX,FACTOR ,NOSHAD,GAMMA
       DIMENSION X(6),Y(10,4)
       IF(COMP-ABS(X(3)))  1,2,2
1      Y(N,1) = X(1)
       Y(N,2) = X(2)
       Y(N,3) = X(3)
       Y(N,4) = X(4)
       GO TO 999
2      IF((H6.LE.H7).AND.(H7.LE.H5))GO TO 20
       IF((H6.LE.H5).AND.(H5.LE.H7))GO TO 21
       IF((H7.LT.H6).AND.(H6.LE.H5))GO TO 22
       GO TO 998
20     IF(H5.GE.X(5))GO TO 23
       Y(N,2) = .5*(X(5)+H5)
       Y(N,4)=X(4)*(X(5)-H5)/X(5)
       GO TO 24
21     IF(H7.GE.X(5))GO TO 23
       Y(N,2) = .5*(X(5)+H7)
       Y(N,4) =(X(4)*(X(5)-H7))/X(5)
       GO TO 24
22     IF(H5.GE.X(5)) GO TO 25
       Y(N,4)=X(5)-H5+H6-H7
       Y(N,2) =  (H6**2 - H7**2 +X(5)**2- H5**2)/(2.*Y(N,4))
       Y(N,4)=X(4)*Y(N,4)/X(5)
       GO TO 24
25     IF (H6.GE.X(5)) GO TO 21
       Y(N,2) =(H7+H6)/2.
       Y(N,4) =(H7-H6)*X(4)/X(5)
24     Y(N,1)=X(1)
       Y(N,3)=X(3)
       GO TO 997
998    WRITE(6,3)H5,H6,H7,N
3      FORMAT(1H , 13HERROR IN FLIM   3E20.5,I5)
23     Y(N,4)=0.0
997    Y(N,2)=Y(N,2)-X(5)*.5 +ABS(X(2))
       IF (X(2))30,31,31
30     Y(N,2)=-Y(N,2)
999    CONTINUE
31     RETURN
       END
```

```
                         SAMPLE DATA LISTING
NOSHAD,NEGO,NDAYS              1     0    15
NO. ORBITS 5.7740563
IAIR,ISUN,IGRAV
ITORTA-EGO                     2
F4Y
F4X
CANT            17.5     -98.5        40.5       344        51.0
S2
C3
C1
BX            -195.34                          160.0      360.0
 COPEP      0.0         43.5                      200.0
 OPEP       0.0         43.5                  332.0       39.9
BOOM6       -5.6       -163.9       -19.2      358.0      238.7
SPHERE6     -5.6       -288.7       -18.0       133.1      13.08     238.7
BOOM5       11.1        152.8       -24.0      366.5      258.6
CS-TORUS    11.1        347.1       -24.2      342.0      114.0      266.6
BOX-X5      11.1        285.7       -24.2       81.0        8.0      258.6
BOX-Y5      11.1        277.7       -24.2       81.0        8.0
H,EABCL          70.0       90.0        30.66      31.67     67.0       23.5
W,SSMAS,Y6   10.0       0.8         0.8        1.03
XG,YG,ZG         .41    1.03        -0.14
Y,Z,OPEP,PAD            .78          .78        .73        .8
B6,BX,B5,F5              .1           .1         .1         .75
       ACS               .78
 OP         155.0       166.0       83.5
1 ATMD                   5.5E-13               4.0E-14               4.5E-15
2 ATMD                   5.5E-16               1.0E-16               2.8E-17
3 ATMD                   1.0E-17               4.0E-18               1.6E-18
4 ATMD                   7.0E-19               3.0E-19               1.4E-19
5 ATMD                   1.0E-19               7.0E-20
V                        .94E-07
1-THRSTX     .13         .15          .17        .185       .20         .225
2-THRSTX     .24         .25          .26        .270       .285        .30
3-THRSTX     .305        .31          .32        .326       .3233       .324
4-THRSTX     .324
1-THRSTY     .324        .33          .34        .345       .348        .3499
2-THRSTY     .38         .351         .35        .349       .345        .324
3-THRSTY     .32         .31          .3         .275       .25         .225
4-THRSTY     .13
XXI,YYI,ZZI             568.8       393.3      965.7
GTHX,GTHY,GTHZ          -0.89       -0.74      -0.58
NORBIT,NINTER,IPRNT      25   360     2
ERRREP                   .0001
GM,RE                    1.408E+16   .20902913E+08
EGO A   I-1               .26169836E09
EGO E,XI,S   I-1         .916668     49.36      45.03
EGO OMEGA,BETA   I-1    -45.35       30.47
EGO ALPHAS       I-1     314.5       315.0      334.0      334.5
EGO A   I-2               .26169836E09
EGO E,XI,S   I-2         .914091     49.51      31.99
EGO OMEGA,BETA   I-2    -44.49       31.99
EGO ALPHAS       I-2     315.0       315.5      341.0      341.5
EGO A   I-3               .26169833E09
EGO E,XI,S   I-3         .911795     49.66      17.45
EGO OMEGA,BETA   I-3    -42.81       33.28
EGO ALPHAS       I-3     322.5       323.0      346.0      346.5
EGO A   I-4               .26169830E09
```

EGO E,XI,S	I-4	.910249	49.78	-10.12	
EGO OMEGA,BETA	I-4	-41.26	35.37		
EGO ALPHAS	I-4	331.5	332.0	349.0	349.5
EGO A	I-5	.26189826E09			
EGO E,XI,S	I-5	.909503	49.94	-11.01	
EGO OMEGA,BETA	I-5	-39.68	35.93		
EGO ALPHAS	I-5	340.5	341.0	352.0	352.5
EGO A	I-6	.26189823E09			
EGO E,XI,S	I-6	.909355	50.23	-29.28	
EGO OMEGA,BETA	I-6	-37.84	37.65		
EGO ALPHAS	I-6	346.5	347.0	356.0	356.5
EGO A	I-7	.26189820E09			
EGO E,XI,S	I-7	.909382	50.60	-38.10	
EGO OMEGA,BETA	I-7	-35.96	39.50		
EGO ALPHAS	I-7	350.0	350.5	0.0	0.5
EGO A	I-8	.26189817E09			
EGO E,XI,S	I-8	.909048	51.04	-38.67	
EGO OMEGA,BETA	I-8	-34.01	41.46		
EGO ALPHAS	I-8	351.5	352.0	4.0	4.5
EGO A	I-9	.26189813E09			
EGO E,XI,S	I-9	.907907	51.74	-9.52	
EGO OMEGA,BETA	I-9	-32.12	43.75		
EGO ALPHAS	I-9	353.5	354.0	8.0	8.5
EGO A	I-10	.26189810E09			
EGO E,XI,S	I-10	.905704	52.35	2.17	
EGO OMEGA,BETA	I-10	-30.42	45.62		
EGO ALPHAS	I-10	354.5	355.0	13.0	13.5
EGO A	I-11	.26189807E09			
EGO E,XI,S	I-11	.902477	52.90	19.25	
EGO OMEGA,BETA	I-11	-29.02	47.29		
EGO ALPHAS	I-11	356.5	357.0	17.0	17.5
EGO A	I-12	.26189800E09			
EGO E,XI,S	I-12	.898508	53.32	36.64	
EGO OMEGA,BETA	I-12	-27.92	48.63		
EGO ALPHAS	I-12	358.5	359.0	20.0	20.5
EGO A	I-13	.26189797E09			
EGO E,XI,S	I-13	.894287	53.59	52.16	
EGO OMEGA,BETA	I-13	-27.10	49.68		
EGO ALPHAS	I-13	1.5	2.0	23.0	23.5
EGO A	I-14	.26189794E09			
EGO E,XI,S	I-14	.890350	53.76	67.96	
EGO OMEGA,BETA	I-14	-26.47	50.51		
EGO ALPHAS	I-14	5.5	6.0	25.0	25.5
EGO A	I-15	.26189790E09			
EGO E,XI,S	I-15	.887137	53.89	82.60	
EGO OMEGA,BETA	I-15	-25.96	51.26		
EGO ALPHAS	I-15	13.5	14.0	21.0	21.5
EGO A	I-16	.26189787E09			
EGO E,XI,S	I-16	.884854	54.06	100.06	
EGO OMEGA,BETA	I-16	-25.46	52.06		
EGO ALPHAS	I-16				
EGO A	I-17	.26189784E09			
EGO E,XI,S	I-17	.883441	54.30	119.19	
EGO OMEGA,BETA	I-17	-24.90	52.99		
EGO ALPHAS	I-17				
EGO A	I-18	.26189780E09			
EGO E,XI,S	I-18	.882455	54.71	101.65	
EGO OMEGA,BETA	I-18	-24.25	54.15		
EGO ALPHAS	I-18				
EGO A	I-19	.26189774E09			
EGO E,XI,S	I-19	.881477	55.44	100.52	
EGO OMEGA,BETA	I-19	-23.53	54.98		

```
EGO ALPHAS        I-19
EGO A  I-20                .26189771E09
EGO E,XI,S  I-20         .879962    55.89      113.28
EGO OMEGA,BETA  I-20 -22.77      57.82
EGO ALPHAS        I-20
EGO A  I-21                .26189767E09
EGO E,XI,S  I-21         .877537    56.56      105.71
EGO OMEGA,BETA  I-21 -22.04      58.16
EGO ALPHAS        I-21
EGO A  I-22                .26189764E09
EGO E,XI,S  I-22         .874056    57.19      90.79
EGO OMEGA,BETA  I-22 -21.40      49.35
EGO ALPHAS        I-22
EGO A  I-23                .26189761E09
EGO E,XI,S  I-23         .869629    57.67      79.40
EGO OMEGA,BETA  I-23 -20.87      60.36
EGO ALPHAS        I-23
EGO A  I-24                .26189757E09
EGO E,XI,S  I-24         .864587    58.03      63.63
EGO OMEGA,BETA  I-24 -20.48      61.09
EGO ALPHAS        I-24  287.5     288.0       304.0       304.5
EGO A  I-25                .26189754E09
EGO E,XI,S  I-25         .859395    58.23      49.67
EGO OMEGA,BETA  I-25 -20.22      61.65
EGO ALPHAS        I-25
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BAARINC

APPENDIX C

OGO ORBITAL PARAMETER HISTORIES USED IN GAS BUDGET COMPUTATIONS

APPENDIX C

OGO ORBITAL PARAMETER HISTORIES USED IN GAS BUDGET COMPUTATIONS

Time in Orbit (Days)	POGO, 150 n.m.		POGO, 155 n.m.		POGO, 180 n.m.		POGO, 200 n.m.		EGO	
	Semimajor Axis (Ft. x 10^7)	Eccentricity	Semimajor Axis (Ft. x 10^7)	Eccentricity	Semimajor Axis (Ft. x 10^7)	Eccentricity	Semimajor Axis (Ft. x 10^7)	Eccentricity	Semimajor Axis (Kms)	Eccentricity
0	2.2885	0.0462*	2.2920	0.0457	2.2996	0.0423	2.3057	0.0395	79286.8	0.91667
15	2.2865		2.2913	0.0463	2.2993	0.0430	2.3055	0.0403	79286.8	0.91409
30	2.2853		2.2895	0.0461	2.2986	0.0433	2.3052	0.0407	79286.8	0.91180
45	2.2853		2.2854	0.0444	2.2970	0.0425	2.3044	0.0403	79286.8	0.91025
60	2.2865		2.2817	0.0422	2.2951	0.0410	2.3034	0.0392	79286.8	0.90950
75	2.2885		2.2795	0.0407	2.2941	0.0399	2.3028	0.0382	79286.8	0.90936
90	2.2878		2.2771	0.0396	2.2932	0.0394	2.3024	0.0378	79286.7	0.90938
105	2.2865		2.2749	0.0394	2.2924	0.0397	2.3021	0.0382	79286.7	0.90905
120	2.2846		2.2731	0.0395	2.2916	0.0402	2.3017	0.0389	79286.7	0.90791
135	2.2840		2.2710	0.0391	2.2906	0.0403	2.3012	0.0392	79286.7	0.90570
150	2.2833		2.2687	0.0377	2.2897	0.0396	2.3007	0.0388	79286.7	0.90248
165	2.2840		2.2659	0.0358	2.2889	0.0385	2.3004	0.0380	79286.7	0.89851
180	2.2853		2.2623	0.0338	2.2879	0.0374	2.3000	0.0371	79286.7	0.89429
195	2.2865		2.2569	0.0321	2.2862	0.0368	2.2992	0.0367	79286.7	0.89035
210	2.2846		2.2543	0.0320	2.2849	0.0371	2.2985	0.0370	79286.7	0.88714
225	2.2814		2.2521	0.0319	2.2841	0.0376	2.2981	0.0377	79286.7	0.88485
240	2.2795		2.2500	0.0310	2.2834	0.0376	2.2978	0.0380	79286.6	0.88341
255	2.2789		2.2475	0.0292	2.2835	0.0368	2.2974	0.0376	79286.6	0.88246
270	2.2802		2.2434	0.0268	2.2812	0.0354	2.2969	0.0366	79286.6	0.88148
285	2.2808		2.2383	0.0248	2.2793	0.0341	2.2959	0.0355	79286.6	0.88000
300	2.2776		2.2326	0.0236	2.2779	0.0339	2.2951	0.0352	79286.6	0.87754
315	2.2668		2.2244	0.0215	2.2768	0.0343	2.2947	0.0357	79286.6	0.87406
330			2.2147	0.0181	2.2753	0.0345	2.2942	0.0364	79286.6	0.86963
345			2.2096	0.0156	2.2738	0.0339	2.2935	0.0364	79286.6	0.86459
360			2.2059	0.0136	2.2730	0.0329	2.2931	0.0359	79286.5	0.85940

* Subsequent eccentricities unavailable